# A New 4-D Multi-Stable Hyperchaotic Two-Scroll System with No-Equilibrium and its Hyperchaos Synchronization

# A New 4-D Multi-Stable Hyperchaotic Two-Scroll System with No-Equilibrium and its Hyperchaos Synchronization

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**Abstract.** A new 4-D multi-stable hyperchaotic two-scroll system with four quadratic nonlinearities is proposed in this paper. The dynamical properties of the new hyperchaotic system are described in terms of finding equilibrium points, phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. We discover that the new hyperchaotic system has no equilibrium point and hence it exhibits a hidden attractor. Furthermore, we show that the new hyperchaos system has multi-stability by the coexistence of hyperchaotic attractors for different values of initial conditions. As a control application, we use integral sliding mode control (ISMC) to derive new results for the hyperchaos synchronization of the new 4-D multi-stable hyperchaotic two-scroll system with hidden attractor.

## 1. Introduction

Chaos theory deals with nonlinear dynamical systems exhibiting high sensitivity to small changes in initial conditions [1-2]. Mathematically, chaotic systems are characterized by the presence of at least one positive Lyapunov exponent. Chaotic systems are very useful in many applications in science and engineering such as weather systems [3-5], ecology [6-10], neurons [11-12], biology [13-16], cellular neural networks [17-18], chemical reactors [19-24], brain waves [25-26], Tokamak systems [27-28], oscillators [29-35], encryption [36-44], finance systems [45-46], circuits [47-50], etc.

Hyperchaotic systems are defined as nonlinear dynamical systems having two or more positive Lyapunov exponents [1-2]. They exhibit more complex behaviour than chaotic dynamical systems as the trajectories of hyperchaotic systems can expand in two different directions corresponding to the two positive Lyapunov exponents. Many new hyperchaotic systems with special behaviour have been reported in the literature such as hyperchaotic Lorenz system [51], hyperchaotic Chen system [52], hyperchaotic Lü system [53], hyperchaotic Vaidyanathan systems [54-55], etc.

In this work, we report a new hyperchaotic two-scroll system with no equilibrium point. Thus, the new hyperchaos system belongs to the new class of hyperchaotic systems with *hidden attractors* [2].

We show that the new 4-D hyperchaotic system exhibits a two-scroll attractor. We analyze the dynamical properties of the new hyperchaotic two-scroll system with phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc.

Section 2 describes the dynamics of the new hyperchaotic two-wing system, its phase plots and Lyapunov exponents. Section 3 describes the dynamic analysis of the new hyperchaotic two-wing system. We show that the new hyperchaos system exhibits multi-stability and this is confirmed b with the coexistence of two different hyperchaotic attractors for different initial conditions. Section 4 describes the hyperchaos synchronization of the new hyperchaos systems using sliding mode control. Section 5 draws the main conclusions of this research work.

## 2. A New Hyperchaotic Two-Scroll system with No Equilibrium Point

In this work, we report a new 4-D dynamical system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + x_4 \\ \dot{x}_2 = bx_2 - cx_1 x_3 - px_1^2 + x_4 \\ \dot{x}_3 = x_1 x_2 - d \\ \dot{x}_4 = -x_1 - x_2 \end{cases} \tag{1}$$

where $X = (x_1, x_2, x_3, x_4)$ is the state and $a, b, c, d, p$ are positive constants.

In this paper, we show that the 4-D system (1) is *hyperchaotic* for the parameter values

$$a = 16,\ b = 3,\ c = 8,\ d = 20,\ p = 0.1 \tag{2}$$

Using Wolf's algorithm [56], the Lyapunov exponents of the system (1) for the parameter set $(a, b, c, d, p) = (16, 3, 8, 20, 0.1)$ and the initial state $X(0) = (0.2, 0.2, 0.2, 0.2)$ were found as

$$LE_1 = 3.0085,\ LE_2 = 0.0642,\ LE_3 = 0,\ LE_4 = -16.0506 \tag{3}$$

Thus, the 4-D system (1) is hyperchaotic with two positive Lyapunov exponents.

The high value of $LE_1$ over 3 indicates the highly complex chaotic nature of the new 4-D hyperchaotic system (1).

It is noted that the sum of the Lyapunov exponents in (3) is negative.

$$LE_1 + LE_2 + LE_3 + LE_4 \approx -13 < 0 \tag{4}$$

This shows that the system (1) is dissipative with a hyperchaotic attractor.

The Kaplan-Yorke dimension of the system (1) is computed as

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.1914 \tag{5}$$

Figure 1 shows the Lyapunov exponents of the 4-D dissipative hyperchaotic system (1) for the parameter set $(a, b, c, d, p) = (16, 3, 8, 20, 0.1)$ and initial state $X(0) = (0.2, 0.2, 0.2, 0.2)$.

Figures 2-5 show the 2-D phase portraits of the hyperchaotic system (1) for the parameter set $(a, b, c, d, p) = (16, 3, 8, 20, 0.1)$ and initial state $X(0) = (0.2, 0.2, 0.2, 0.2)$. From the phase plots, we see that the 4-D system (1) has a *hyperchaotic two-scroll attractor*.

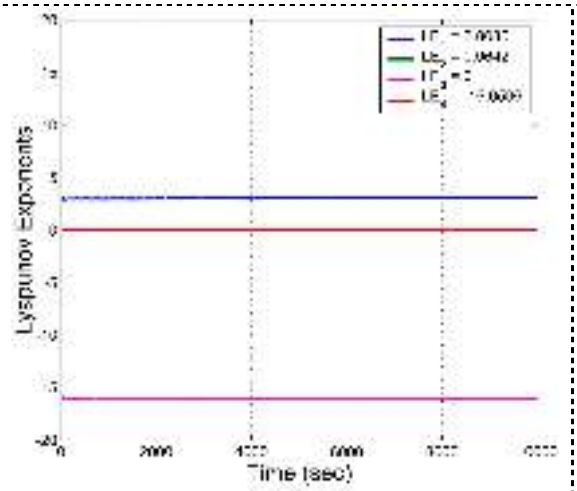


**Figure 1.** Lyapunov exponents of the hyperchaotic two-scroll system (1) for the parameter set $(a,b,c,d,p)=(16,3,8,20,0.1)$ and initial state $X(0)=(0.2,0.2,0.2,0.2)$

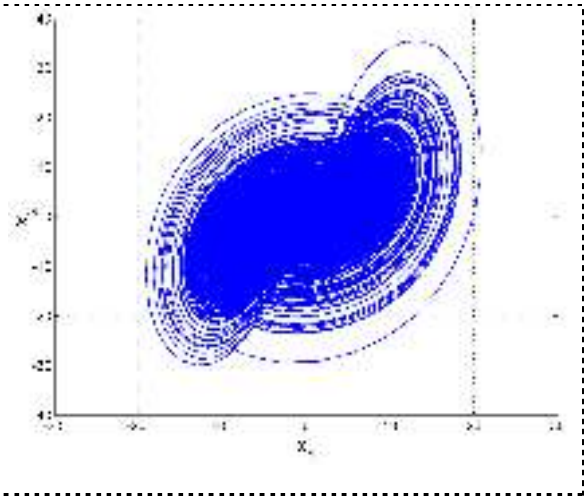

**Figure 2.** MATLAB plot showing the 2-D phase portrait of the hyperchaotic two-wing system (1) in the $(x_1,x_2)-$plane for $(a,b,c,d,p)=(16,3,8,20,0.1)$ and $X(0)=(0.2,0.2,0.2,0.2)$

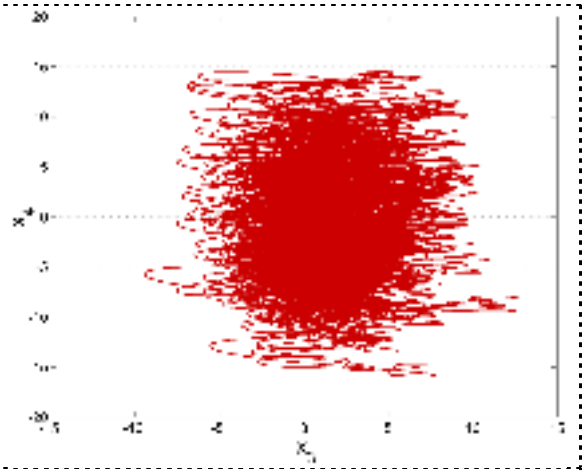

**Figure 3.** MATLAB plot showing the 2-D phase portrait of the hyperchaotic two-wing system (1) in the $(x_2,x_3)-$plane for $(a,b,c,d,p)=(16,3,8,20,0.1)$ and $X(0)=(0.2,0.2,0.2,0.2)$

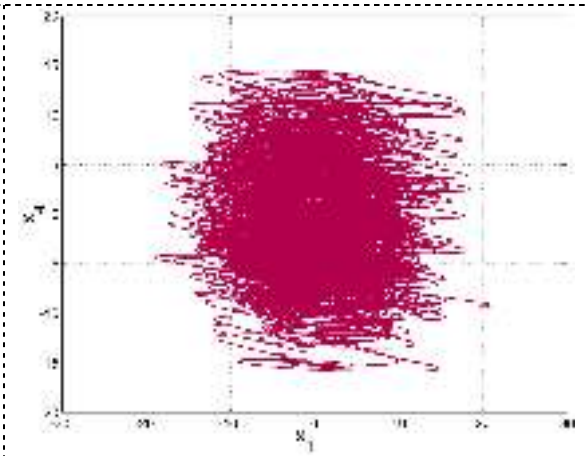

**Figure 4.** MATLAB plot showing the 2-D phase portrait of the hyperchaotic two-wing system (1) in the $(x_3,x_4)-$plane for $(a,b,c,d,p)=(16,3,8,20,0.1)$ and $X(0)=(0.2,0.2,0.2,0.2)$

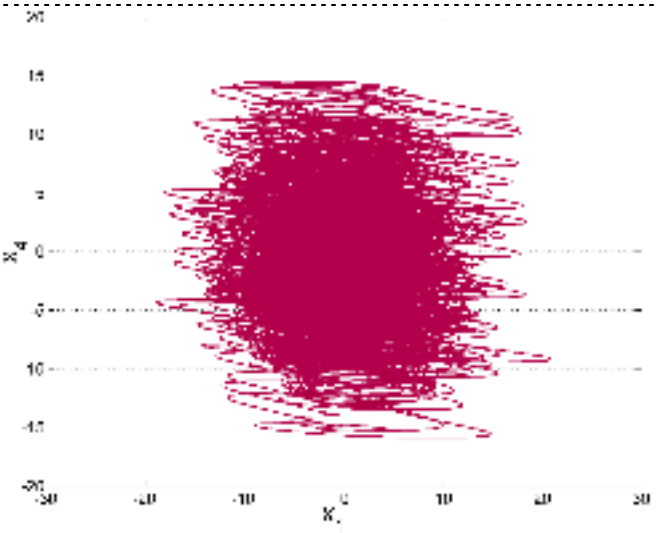

**Figure 5.** MATLAB plot showing the 2-D phase portrait of the hyperchaotic two-wing system (1) in the $(x_1, x_4)-$plane for $(a,b,c,d,p)=(16,3,8,20,0.1)$ and $X(0)=(0.2,0.2,0.2,0.2)$

The equilibrium points of the new hyperchaotic system (1) are obtained by solving the system

$$a(x_2-x_1)+x_2x_3+x_4=0 \tag{6a}$$

$$bx_2-cx_1x_3-px_1^2+x_4=0 \tag{6b}$$

$$x_1x_2-d=0 \tag{6c}$$

$$-x_1-x_2=0 \tag{6d}$$

From (6d), we deduce that $x_1=-x_2$.

Substituting above in (6c), we get $-x_2^2-d=0$ or

$$x_2^2=-d \tag{7}$$

Since $d>0$, the equation (7) does not admit any real solution.

Thus, the new 4-D hyperchaotic two-scroll system does not have any equilibrium point.
Hence, we conclude that the 4-D hyperchaotic system (1) has *hidden* attractor [2].

## 3. Dynamic Analysis for the New Hyperchaotic System

### *3.1 Dissipativity*

The 4-D hyperchaotic two-scroll system introduced in this work is given by the dynamics

$$\begin{cases}\dot{x}_1=a(x_2-x_1)+x_2x_3+x_4=f_1(x_1,x_2,x_3,x_4)\\ \dot{x}_2=bx_2-cx_1x_3-px_1^2+x_4=f_2(x_1,x_2,x_3,x_4)\\ \dot{x}_3=x_1x_2-d=f_3(x_1,x_2,x_3,x_4)\\ \dot{x}_4=-x_1-x_2=f_4(x_1,x_2,x_3,x_4)\end{cases} \tag{8}$$

The divergence of the flow defined by the system (8) is

$$\frac{\partial f_1}{\partial x_1}+\frac{\partial f_2}{\partial x_2}+\frac{\partial f_3}{\partial x_3}+\frac{\partial f_4}{\partial x_4}=-(a-b), \tag{9}$$

which is negative for the chosen parameter values (2).

This show that the 4-D hyperchaotic system (8) is dissipative.

Hence, the trajectories of the 4-D system (8) evolve to lie within a bounded region of the phase space.

*3.2 Multi-stability*

Multistability means the coexistence of two or more attractors under different initial conditions but with the same parameter set. It is an interesting phenomenon and can usually be found in many nonlinear systems [2]. Multistability can lead to very complex behaviors in a dynamical system [2].

It is interesting that our system (1) can exhibit coexisting attractors when choosing different initial conditions. For example, when selecting $(a,b,c,d,p)=(16,3,8,20,0.1)$ and the initial conditions $X_0=(0.2,0.2,0.2,0.2)$ (blue) and $Y_0=(0.2,-0.2,0.2,-0.2)$ (red), the 4-D hyperchaotic two-scroll system (1) displays coexisting hyperchaotic attractor (blue) and hyperchaotic attractor (red) as illustrated in Figures 6 and 7 respectively.

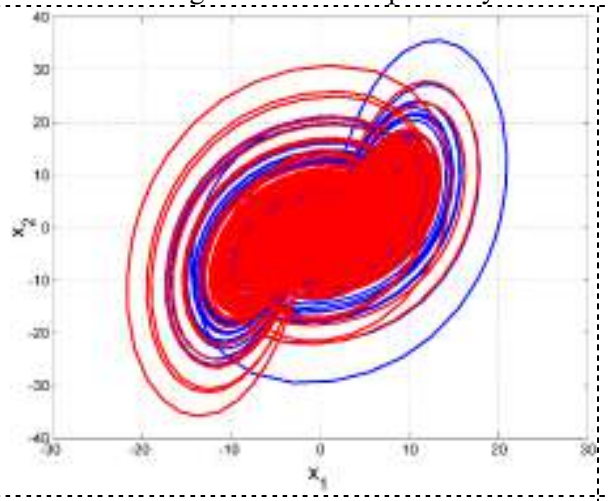

**Figure 6.** Multi-stability of the hyperchaotic system (1): Coexisting hyperchaotic attractors for $(a,b,c,d,p)=(16,3,8,20,0.1)$ in the $(x_1,x_2)-$plane. Blue color shows the hyperchaotic attractor of (1) with initial state $X_0=(0.2,0.2,0.2,0.2)$ and red color shows the hyperchaotic attractor of (1) with the initial state $Y_0=(0.2,-0.2,0.2,-0.2)$

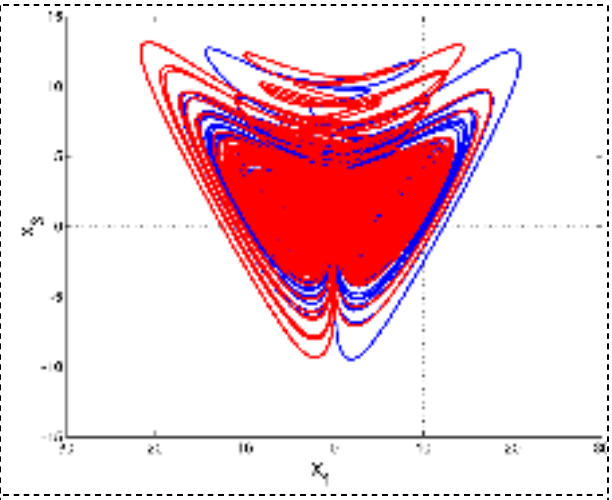

**Figure 7.** Multi-stability of the hyperchaotic system (1): Coexisting hyperchaotic attractors for $(a,b,c,d,p)=(16,3,8,20,0.1)$ in the $(x_1,x_3)-$plane. Blue color shows the hyperchaotic attractor of (1) with initial state $X_0=(0.2,0.2,0.2,0.2)$ and red color shows the hyperchaotic attractor of (1) with the initial state $Y_0=(0.2,-0.2,0.2,-0.2)$

## 4. Hyperchaos Synchronization of the New Hyperchaotic Two-Scroll System via Integral Sliding Mode Control

This section derives new results for the hyperchaos synchronization of a pair of new hyperchaotic systems taken as master and slave systems using integral sliding mode control [2].

As the master system, we consider the new hyperchaotic two-scroll system given by

$$\begin{cases}\dot{x}_1=a(x_2-x_1)+x_2x_3+x_4\\ \dot{x}_2=bx_2-cx_1x_3-px_1^2+x_4\\ \dot{x}_3=x_1x_2-d\\ \dot{x}_4=-x_1-x_2\end{cases} \tag{10}$$

where $x_1, x_2, x_3, x_4$ are the states and $a, b, c, d, p$ are system parameters.

As the slave system, we consider the new hyperchaotic two-scroll system with controls given by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + y_2 y_3 + y_4 + u_1 \\ \dot{y}_2 = b y_2 - c y_1 y_3 - p y_1^2 + y_4 + u_2 \\ \dot{y}_3 = y_1 y_2 - d + u_3 \\ \dot{y}_4 = -y_1 - y_2 + u_4 \end{cases} \tag{11}$$

where $y_1, y_2, y_3, y_4$ are the states and $u_1, u_2, u_3, u_4$ are the sliding controls to be found.

The synchronization error between the new hyperchaotic systems (10) and (11) is defined as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \\ e_4 = y_4 - x_4 \end{cases} \tag{12}$$

Then we obtain the error dynamics as follows:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 = b e_2 + e_4 - c(y_1 y_3 - x_1 x_3) - p(y_1^2 - x_1^2) + u_2 \\ \dot{e}_3 = y_1 y_2 - x_1 x_2 + u_3 \\ \dot{e}_4 = -e_1 - e_2 + u_4 \end{cases} \tag{13}$$

Based on the sliding mode control theory, the integral sliding surface of each error variable is defined as follows:

$$\begin{cases} s_1 = \left[\frac{d}{dt} + \lambda_1\right]\left[\int_0^t e_1(\tau)d\tau\right] = e_1 + \lambda_1 \int_0^t e_1(\tau)d\tau \\ s_2 = \left[\frac{d}{dt} + \lambda_2\right]\left[\int_0^t e_2(\tau)d\tau\right] = e_2 + \lambda_2 \int_0^t e_2(\tau)d\tau \\ s_3 = \left[\frac{d}{dt} + \lambda_3\right]\left[\int_0^t e_3(\tau)d\tau\right] = e_3 + \lambda_3 \int_0^t e_3(\tau)d\tau \\ s_4 = \left[\frac{d}{dt} + \lambda_4\right]\left[\int_0^t e_4(\tau)d\tau\right] = e_4 + \lambda_4 \int_0^t e_1(\tau)d\tau \end{cases} \tag{14}$$

The derivative of each equation in (14) yields

$$\begin{cases} \dot{s}_1 = \dot{e}_1 + \lambda_1 e_1 \\ \dot{s}_2 = \dot{e}_2 + \lambda_2 e_2 \\ \dot{s}_3 = \dot{e}_3 + \lambda_3 e_3 \\ \dot{s}_4 = \dot{e}_4 + \lambda_4 e_4 \end{cases} \tag{15}$$

The Hurwitz condition is fulfilled if $\lambda_i, (i = 1, 2, 3, 4)$ are positive constants.

Based on the exponential reaching law of sliding mode control theory, we set

$$\begin{cases} \dot{s}_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{s}_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ \dot{s}_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \\ \dot{s}_4 = -\eta_4 \operatorname{sgn}(s_4) - k_4 s_4 \end{cases} \tag{16}$$

Comparing equations (15) and (16), we get

$$\begin{cases} -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 = \dot{e}_1 + \lambda_1 e_1 \\ -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 = \dot{e}_2 + \lambda_2 e_2 \\ -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 = \dot{e}_3 + \lambda_3 e_3 \\ -\eta_4 \operatorname{sgn}(s_4) - k_4 s_4 = \dot{e}_4 + \lambda_4 e_4 \end{cases} \tag{17}$$

Using Eq. (13), we can rewrite Eq. (17) as follows:

$$\begin{cases} -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 = a(e_2 - e_1) + e_4 + y_2 y_3 - x_2 x_3 + u_1 + \lambda_1 e_1 \\ -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 = b e_2 + e_4 - c(y_1 y_3 - x_1 x_3) - p(y_1^2 - x_1^2) + u_2 + \lambda_2 e_2 \\ -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 = y_1 y_2 - x_1 x_2 + u_3 + \lambda_3 e_3 \\ -\eta_4 \operatorname{sgn}(s_4) - k_4 s_4 = -e_1 - e_2 + u_4 + \lambda_4 e_4 \end{cases} \tag{18}$$

From (18), we obtain the required sliding mode control (SMC) laws are as follows:

$$\begin{cases} u_1 = -a(e_2 - e_1) - e_4 - y_2 y_3 + x_2 x_3 - \lambda_1 e_1 - \eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ u_2 = -b e_2 - e_4 + c(y_1 y_3 - x_1 x_3) + p(y_1^2 - x_1^2) - \lambda_2 e_2 - \eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ u_3 = -y_1 y_2 + x_1 x_2 - \lambda_3 e_3 - \eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \\ u_4 = e_1 + e_2 - \lambda_4 e_4 - \eta_4 \operatorname{sgn}(s_4) - k_4 s_4 \end{cases} \tag{19}$$

**Theorem 1.** The integral sliding mode control law (19) renders global hyperchaos synchronization for the new hyperchaotic two-scroll systems (10) and (11) for all initial conditions, where the constants $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \eta_1, \eta_2, \eta_3, \eta_4, k_1, k_2, k_3, k_4$ are all positive.

*Proof.* This result is established with the help of Lyapunov stability theory [1-2].

We take the following quadratic Lyapunov function

$$V(s_1, s_2, s_3, s_4) = \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2 + s_4^2\right), \tag{20}$$

where $s_1, s_2, s_3, s_4$ are defined as in Eq. (14).

It is seen that the quadratic function $V$ is positive definite and radially unbounded on $R^4$.

The time-derivative of (20) is calculated as

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + s_3 \dot{s}_3 + s_4 \dot{s}_4 = \sum_{i=1}^{4} s_i \dot{s}_i \tag{21}$$

Substituting from Eq. (16) into Eq. (21), we get

$$\dot{V} = \sum_{i=1}^{4} s_i [-\eta_i \operatorname{sgn}(s_i) - k_i s_i] = -\sum_{i=1}^{4} \left[ \eta_i \mid s_i \mid + k_i s_i^2 \right] \tag{22}$$

Since $\eta_i > 0$ and $k_i > 0$ for $i = 1, 2, 3, 4,$ we deduce from Eq. (22) that $\dot{V}$ is a negative definite function on $R^4$.

Thus, by Lyapunov stability theory, we conclude that $s_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, 3, 4.$

Hence, we conclude that $e_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, 3, 4.$

This completes the proof. ■

For numerical simulations, we take the initial conditions as in the hyperchaotic case (2), viz. $(a,b,c,d,p)=(16,3,8,20,0.1)$.

We consider the sliding constants as

$$\lambda_i = 0.2,\ \eta_i = 0.2,\ k_i = 22 \text{ for } i = 1,2,3,4 \tag{23}$$

The initial state of the master system (10) is taken as

$$x_1(0)=1.2,\ x_2(0)=3.5,\ x_3(0)=2.4,\ x_4(0)=0.8 \tag{24}$$

The initial state of the slave system (11) is taken as

$$y_1(0)=3.9,\ y_2(0)=1.4,\ y_3(0)=0.3,\ y_4(0)=2.2 \tag{25}$$

Figures 8-11 show the complete synchronization of the new hyperchaotic systems (10) and (11). Figure 12 shows the time-history of the synchronization errors $e_1, e_2, e_3, e_4$.

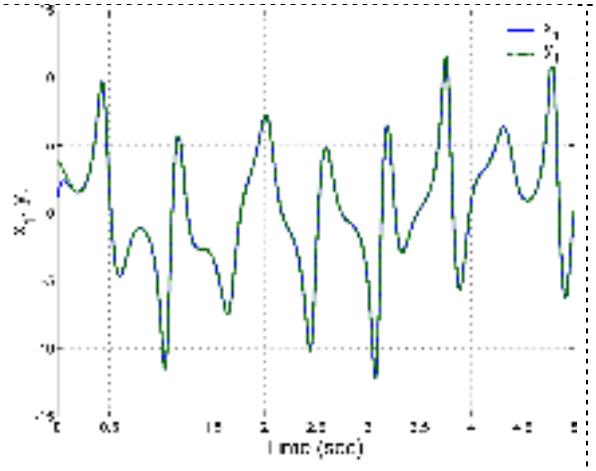

**Figure 8.** Synchronization between the states $x_1$ and $y_1$ of the hyperchaotic systems (10) and (11)

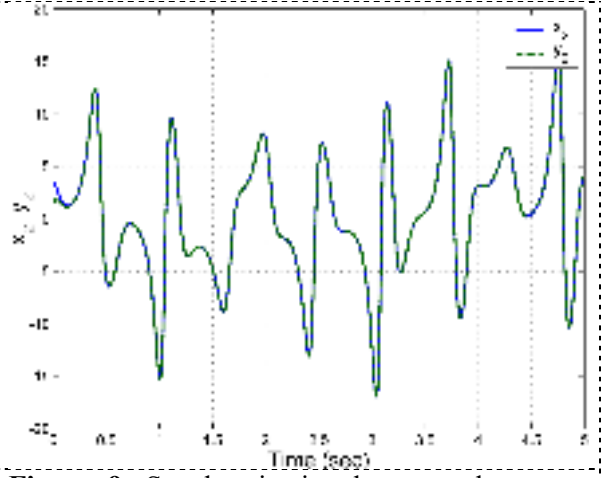

**Figure 9.** Synchronization between the states $x_2$ and $y_2$ of the hyperchaotic systems (10) and (11)

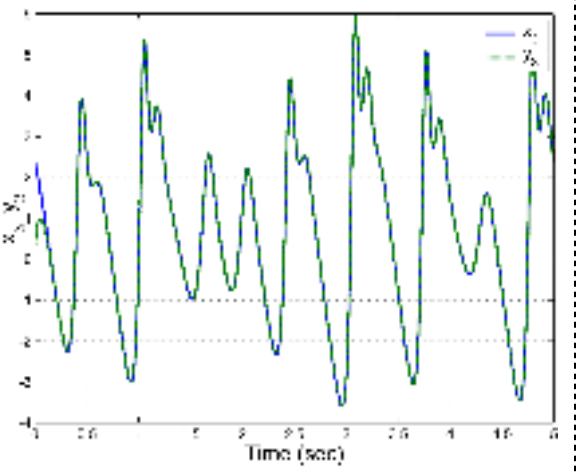

**Figure 10.** Synchronization between the states $x_3$ and $y_3$ of the hyperchaotic systems (10) and (11)

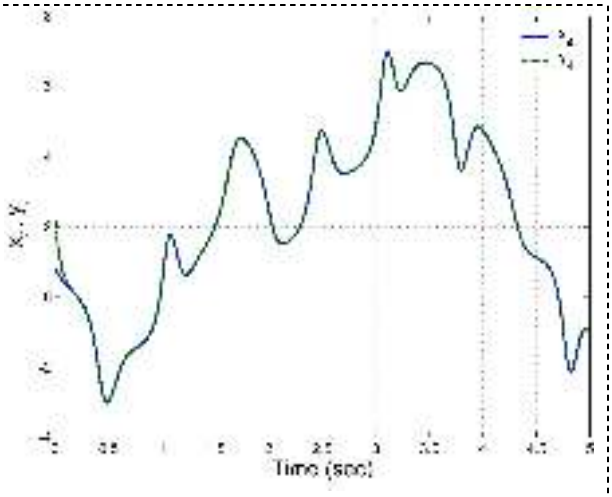

**Figure 11.** Synchronization between the states $x_4$ and $y_4$ of the hyperchaotic systems (10) and (11)

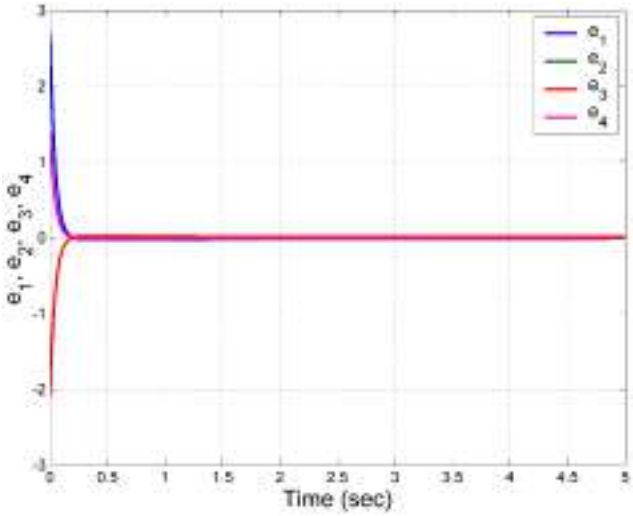


**Figure 12.** Time-history of the synchronization error between the hyperchaotic systems (10) and (11)

## 5. Conclusions

We reported a new 4-D multi-stable hyperchaotic two-scroll system with four quadratic nonlinearities in this paper. The dynamical properties of the new hyperchaotic system were analyzed terms of finding equilibrium points, phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. We showed that the new hyperchaotic system has no equilibrium point and hence it exhibits a hidden attractor. We also demonstrated that the new hyperchaos two-scroll system has multi-stability by the coexistence of hyperchaotic attractors for different values of initial conditions. As a control application, we applied integral sliding mode control to derive new results for the hyperchaos synchronization of the new 4-D multi-stable hyperchaotic two-scroll system.

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