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**Numerical Simulation and Circuit Implementation for a Sprott
Chaotic System with One Hyperbolic
Sinusoidal Nonlinearity**

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Abstract

In this paper, the phenomenon of chaos that produced in the case of Sprott chaotic system with one hyperbolic sinusoidal nonlinearity, have been studied extensively. The basic dynamical properties of such system are discovered through equilibrium points, phase portrait, Lyapunov exponents and Poincaré map. Furthermore, An electronic circuit realization of the proposed system is presented in details. Finally, the circuit experimental results of the chaotic attractors show agreement with numerical simulations.

Keywords: Sprott chaotic system, Lyapunov exponents and Poincaré

Introduction

Chaos is used to explain the behavior of certain dynamical complex, i.e., systems whose state variables evolve with time, which may exhibit dynamics that are highly sensitive to initial conditions. Henri Poincaré was the first discoverer of chaos. In 1890, while studying the three-body problem, he found that there existed some orbits which are non-periodic [1-2]. Interest in nonlinear dynamics and in particular chaotic dynamics has grown rapidly since 1963, Edward Lorenz, the MIT meteorologist, he was designing a 3-D model for weather prediction [3].

In the literature, Sprott (1994) was the first to introduce a simple flow with no equilibrium points [4], In 2000, Malasoma proposed the simplest dissipative jerk equation that is parity invariant [5], Sun and Sprott (2009) constructed a piecewise exponential jerk system [6], In 2010, Sprott gives a 3-D jerk chaotic system having six terms on the R.H.S. with one hyperbolic tangential nonlinearity [7], In 2014, Chunbiao Li and Sprott constructed the chaotic flows with a single nonquadratic term [8] and in 2015, Vaidyanathan created a six-term novel Jerk chaotic system with two exponential nonlinearities [9].

Chaos has been widely applied to many scientific disciplines, such as ecology [10], biology [11], economy [12], random bit generators [13], psychology [14], laser [15], astronomy [16], chemical reaction [17], robotics [18], Text encryption [19], image encryption [20], voice encryption [21], secure communication systems [22-27] etc.

Motivated by the above researches, a Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is proposed in this work, In Section 2, we present Sprott system from three first-order autonomous ODEs, numerical results in evolving phase portraits, Lyapunov exponent's analysis and Poincaré map analysis. In section 3, basic dynamical properties of the Sprott chaotic system are discussed including equilibria and Jacobian matrices. In Section 4 we

present an electronic circuit that implements the nonlinear system and Finally, Section 5 contains the conclusion remarks.

The Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity [28] is described by:

$$\ddot{x} + 0.6\dot{x} + \dot{x} = x - 0.5 \sinh x \quad (1)$$

In system form, the differential equation (1) can be expressed as:

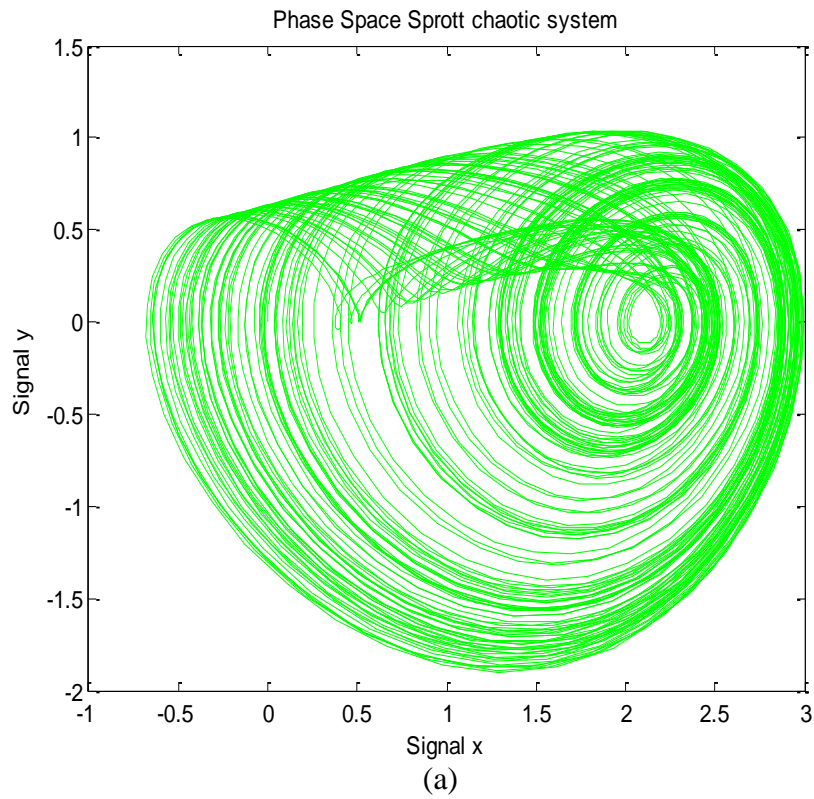
$$\left. \begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= x - a \sinh(x) - y - bz \end{aligned} \right\} \quad (2)$$

Where x, y, z are state variables and when $a = 0.5$ and $b=0.51$, the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2) exhibits strange attractor, we have chosen the initial conditions for the Sprott system (2) as

$$x_0 = 0, y_0 = 0, z_0 = 1 \quad (3)$$

For numerical simulation of chaotic system defined by a set of differential equation such as the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2), different integration techniques can be used. In the MATLAB 2010, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used. Figures 1(a)-(c) show the projections of the phase space orbit on to the x - y plane, the x - z plane and the y - z plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2) presents chaotic attractors of Jerk attractor type.

The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity can be characterized with its Lyapunov exponents which are computed numerically by Wolf algorithm proposed in Ref. [29]. Figure 2 shows the Lyapunov exponents of the Sprott system for constant parameter $a = 0.5$ and $b=0.51$. The Sprott chaotic system is chaotic with a positive Lyapunov exponents. In addition, the Poincare map of the system in Figure 3 also reflects properties of chaos.



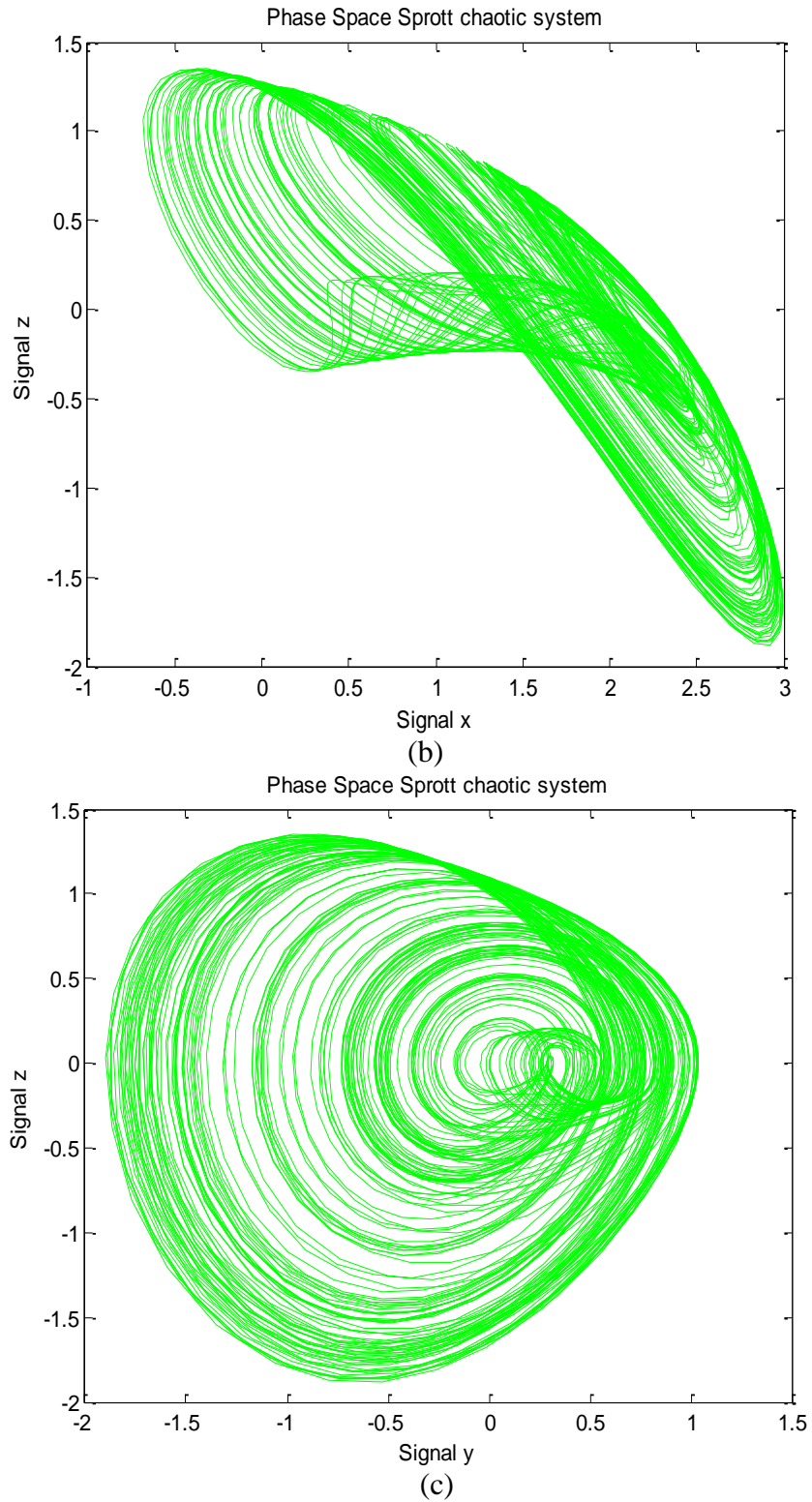


Figure 1 Numerical simulation results using MATLAB 2010, for $a = 0.5$ and $b = 0.51$, in (a) x - y plane, (b) x - z plane, (c) y - z plane.

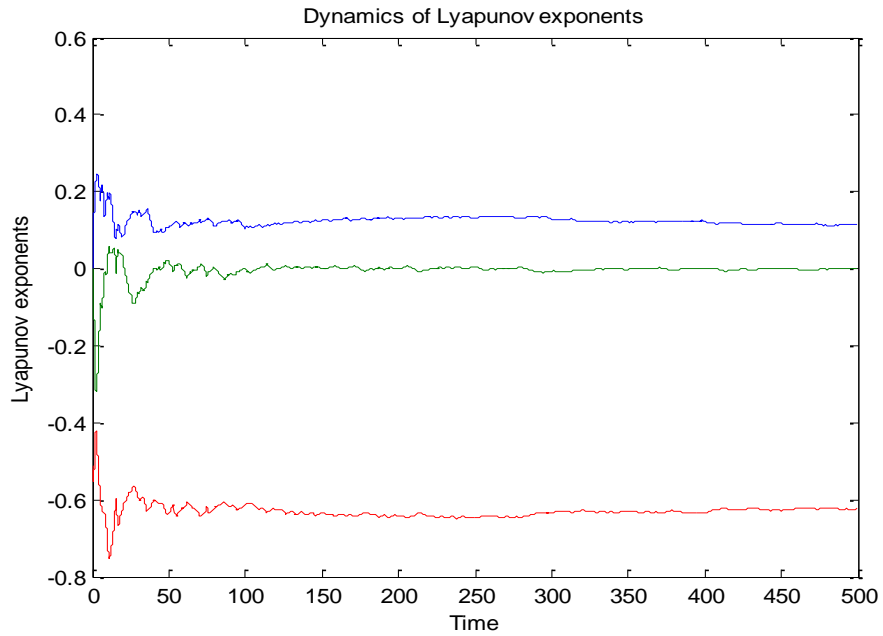


Figure 2 The dynamics of Lyapunov exponents of Sprott chaotic system for $a = 0.5$ and $b = 0.51$, using MATLAB 2010

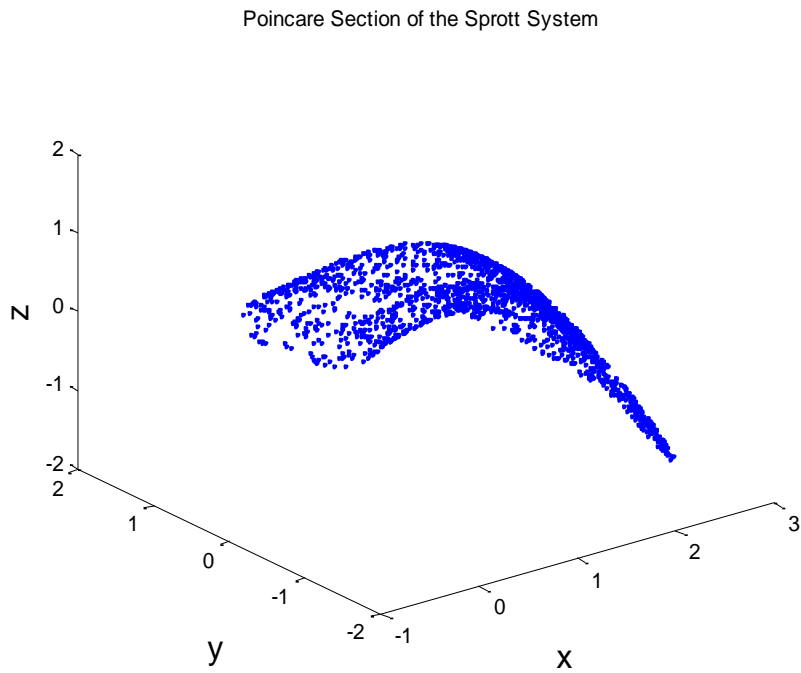


Figure 3 Poincare map in the x - y - z space plane when $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

Basic Properties of the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

The equilibrium points of (2) denote by $E(\bar{x}, \bar{y}, \bar{z})$, are the zeros of its non-linear algebraic system which can be written as:

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The Sprott chaotic system with one hyperbolic sinusoidal nonlinearity has one equilibrium point $E_0(0, 0, 0)$. The dynamical behavior of equilibrium point can be studied by computing the eigenvalue of the Jacobian matrix J of system (2) where:

$$J(\bar{x}, \bar{y}, \bar{z}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 - 0.5 \cosh(x) & -1 & -0.51 \end{bmatrix} \quad (5)$$

For equilibrium points $E_0(0, 0, 0)$, the Jacobian becomes:

$$J(0,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & -1 & -0.51 \end{bmatrix} \quad (6)$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.51\lambda^2 + \lambda - 0.5 \quad (7)$$

Yielding eigenvalues of $\lambda_1 = 0.3753$, $\lambda_2, \lambda_3 = -0.4426 \pm 1.0659 i$, where λ_1 is a positive real number and λ_2, λ_3 are a pair of complex conjugate eigenvalues with negative real parts, which indicates that these two imaginary equilibrium points are saddle points. This equilibrium point is unstable.

Circuit Realization of the the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

In this section, we design an electronic circuit modeling of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity. The circuit in Figure 4 has been designed following an approach based on operational amplifiers [30-34] where the state variables x, y, z of the system (2) are associated with the voltages across the capacitors C_1, C_2 and C_3 , respectively. The nonlinear term of system (2) are implemented with the analog multiplier. By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\left. \begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} y \\ \dot{y} &= \frac{1}{C_2 R_2} z \\ \dot{z} &= \frac{1}{C_3 R_3} x - \frac{1}{C_3 R_4} \sinh(x) - \frac{1}{C_3 R_5} y - \frac{1}{C_3 R_6} z \end{aligned} \right\} \quad (8)$$

We choose $R_1 = R_2 = R_3 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 10 \text{ k}\Omega$, $R_4 = 200\Omega$, $R_6 = 19.61 \text{ k}\Omega$, $R_{14} = 120 \text{ k}\Omega$, $R_{15} = 60 \text{ k}\Omega$, $R_{16} = 1 \text{ M}\Omega$, $C_1 = C_2 = C_3 = 10 \text{ nF}$. The circuit has three integrators by using Op-amp TL082CD in a feedback loop and two multipliers IC AD633. The supplies of all active devices are $\pm 15 \text{ Volt}$. With MultiSIM 10.0, we obtain the experimental observations of system (2) as shown in Figure 5. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Sprott chaotic system is confirmed.

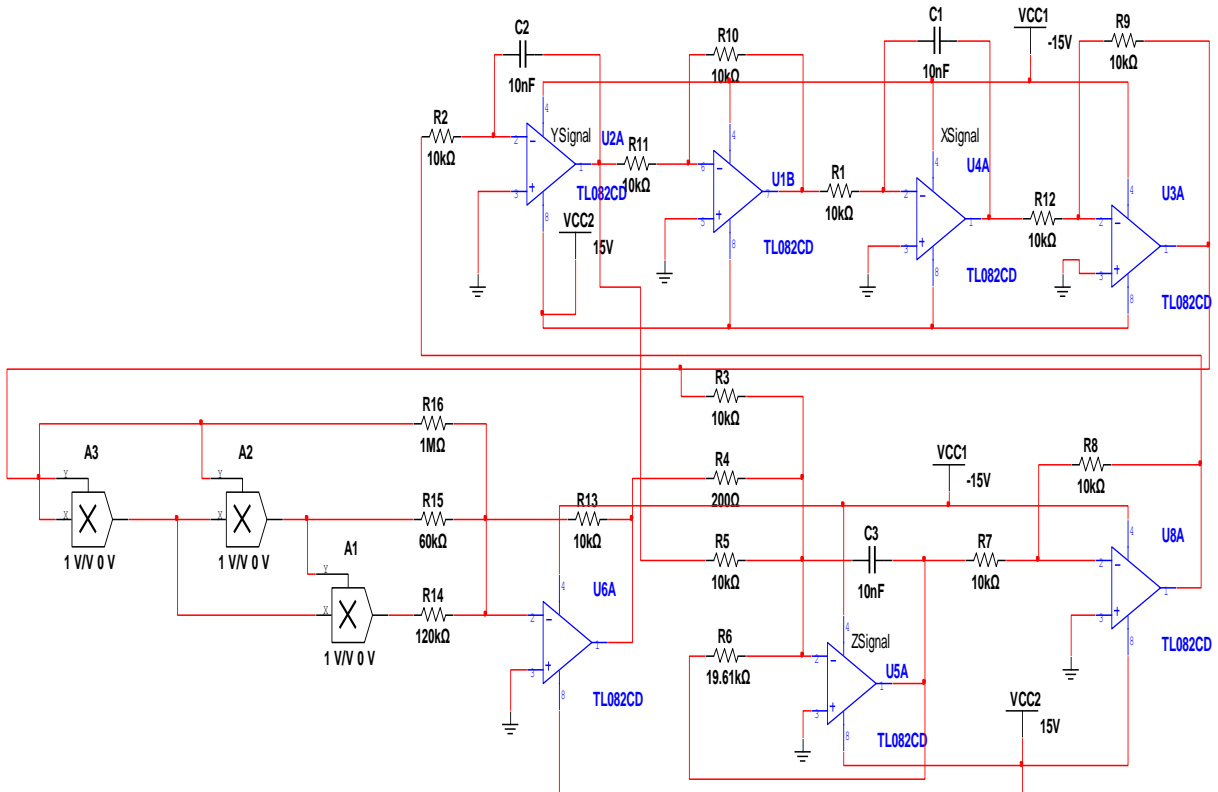
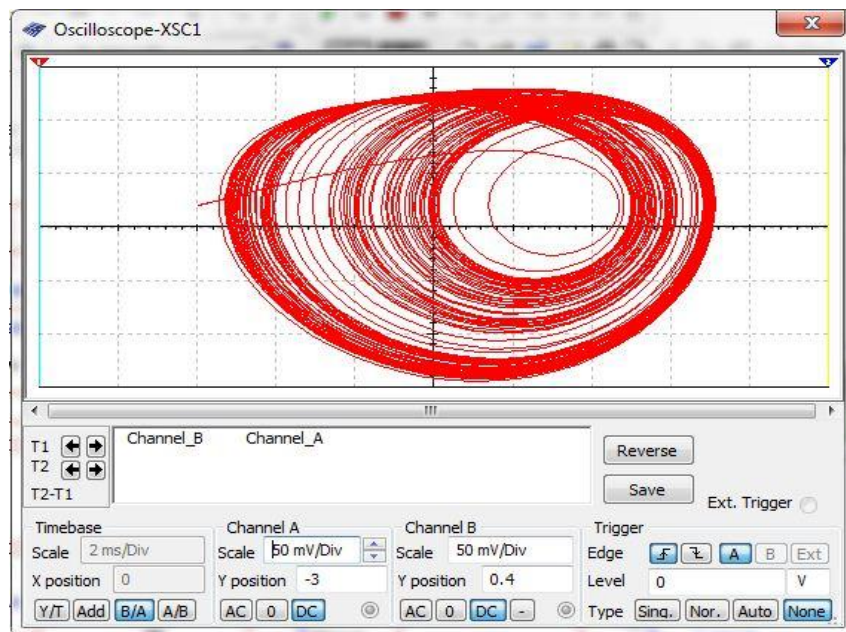
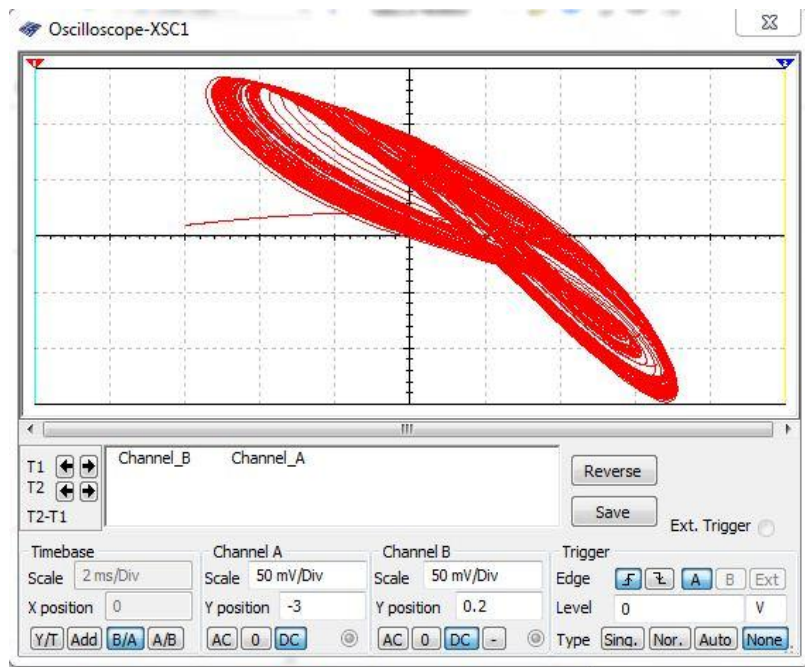


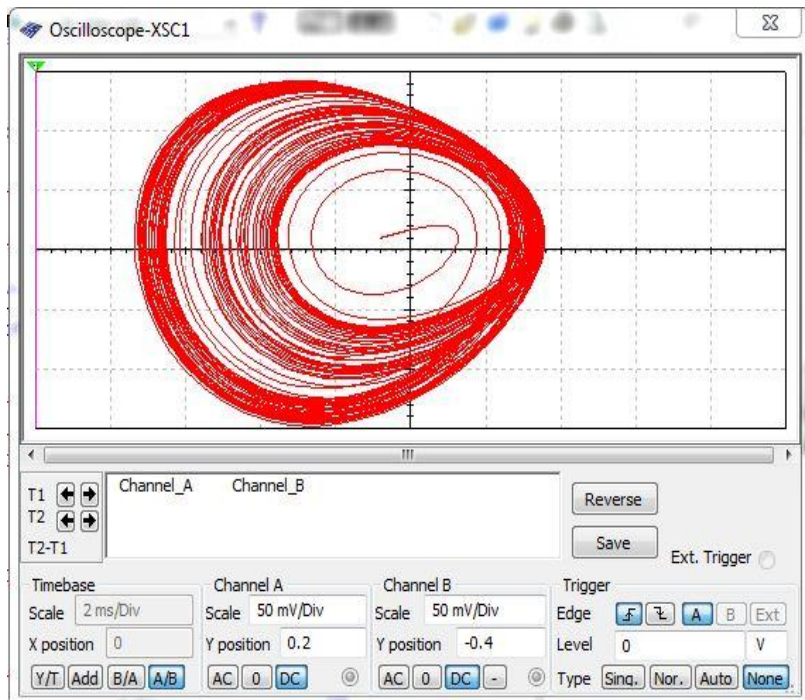
Figure 4 Schematic of the proposed Sprott chaotic system with one hyperbolic sinusoidal nonlinearity by using MultiSIM 10.0.



(a)



(b)



(c)

Figure 5 Various projections of the chaotic attractor using MultiSIM in

x - y plane, (b) x - z plane and (c) y - z plane

Conclusion

A Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is constructed and analyzed. The fundamental properties of the system such as equilibrium points, Lyapunov exponents and Poincaré map as well as its phase portraits were described in detail. Moreover, it is implemented via a designed circuit with MultiSIM and numerical simulation using MATLAB, showing very good agreement with the simulation result. Hence, we can apply this Sprott chaotic system with one hyperbolic sinusoidal nonlinearity in practical applications like robotic, random bits Generator and secure communications

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Title: Numerical simulation and circuit implementation for a Sprott chaotic system with one hyperbolic sinusoidal nonlinearity

Author(s): Aceng Sambas, Mujiarto, Mustafa Mamat and Mada Sanjaya WS

Recommendation: Accept () Revision () Reject ()

Comments:

Chaotic system with one hyperbolic sinusoidal nonlinearity is a special case in nonlinear field. The paper deal with the circuit implementation of this system, which is good and we should encourage this attempt. However, the paper needs more descriptions on how to realize the hyperbolic sinusoidal. And if possible, please try a real experiment not just a MultiSim simulation since the title is numerical simulation and circuit implementation.



NUMERICAL SIMULATION AND CIRCUIT IMPLEMENTATION FOR A SPROTT CHAOTIC SYSTEM WITH ONE HYPERBOLIC SINUSODIAL NONLINEARITY

Aceng Sambas^{a,b,*}, Mujiarto^a, Mustafa Mamat^b and W. S. Mada Sanjaya^c

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2010 Mathematics Subject Classification: **Kindly provide.**

Keywords and phrases: Sprott chaotic system, Lyapunov exponents and Poincaré.

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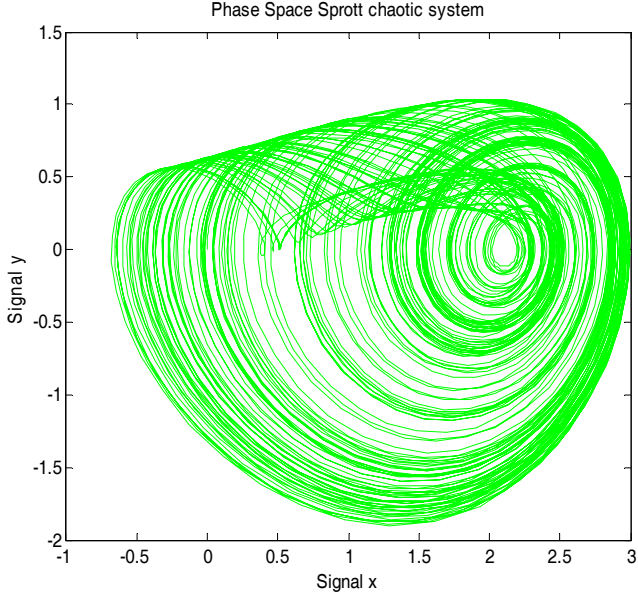
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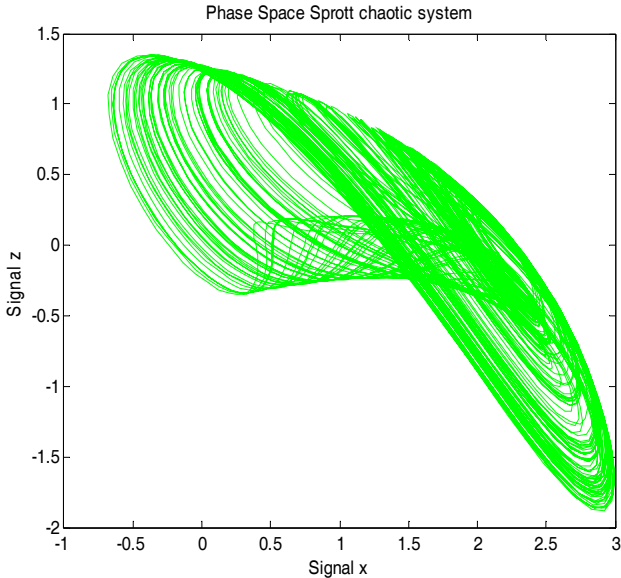
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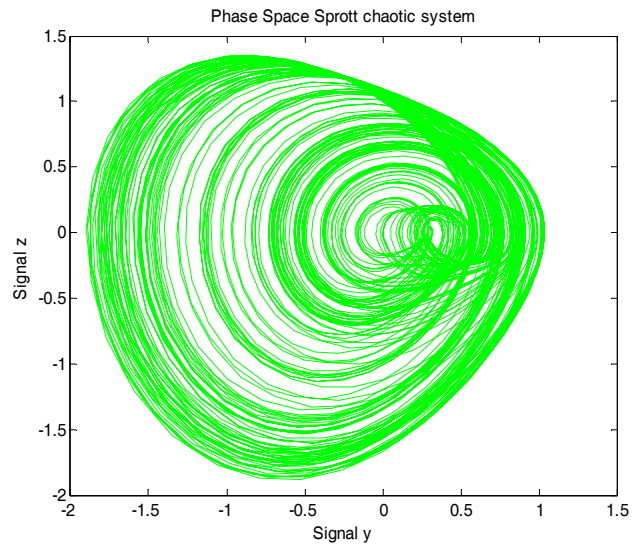
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(a)



(b)



(c)

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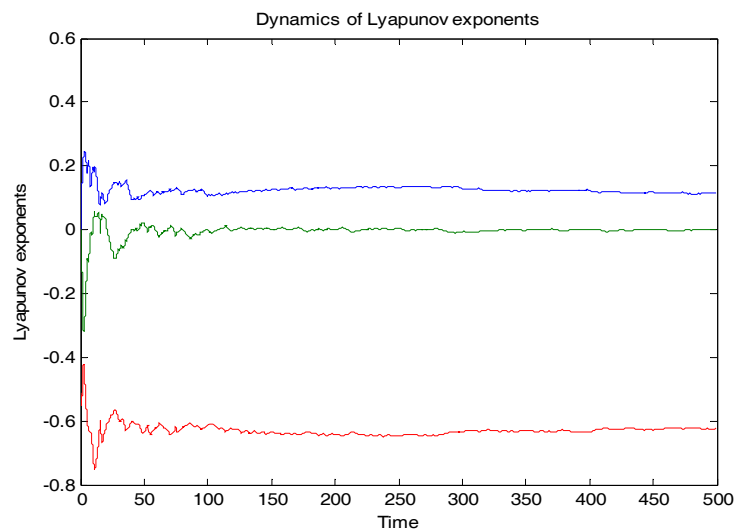


Figure 2. The dynamics of Lyapunov exponents of Sprott chaotic system for $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

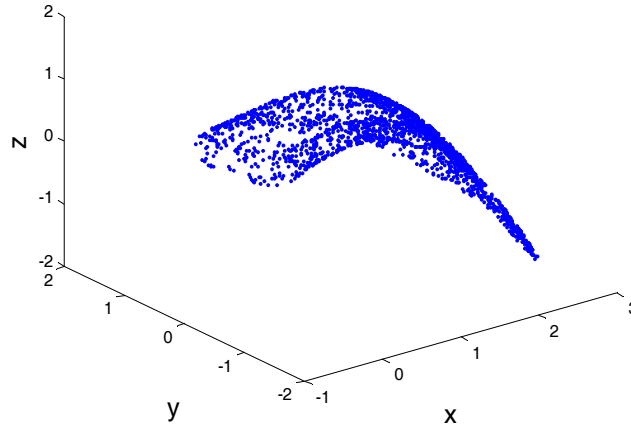


Figure 3. Poincare map in the $x - y - z$ space plane when $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

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The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.51\lambda^2 + \lambda - 0.5. \quad (7)$$

Yielding eigenvalues of $\lambda_1 = 0.3753$, $\lambda_2 \lambda_3 = -0.4426 \pm 1.0659i$, where λ_1 is a positive real number and λ_2, λ_3 are a pair of complex conjugate eigenvalues with negative real parts, which indicates that these two imaginary equilibrium points are saddle points. This equilibrium point is unstable.

Circuit Realization of the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

In this section, we design an electronic circuit modeling of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity. The circuit in Figure 4 has been designed following an approach based on operational amplifiers [30-34], where the state variables x, y, z of the system (2) are associated with the voltages across the capacitors C_1, C_2 and C_3 , respectively. The nonlinear term of system (2) are implemented with the analog multiplier. By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\left. \begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} y \\ \dot{y} &= \frac{1}{C_2 R_2} z \\ \dot{z} &= \frac{1}{C_3 R_3} x - \frac{1}{C_3 R_4} \sinh(x) - \frac{1}{C_3 R_5} y - \frac{1}{C_3 R_6} z \end{aligned} \right\}. \quad (8)$$

We choose $R_1 = R_2 = R_3 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{13} = 10k\Omega$, $R_4 = 200\Omega$, $R_6 = 19.61k\Omega$, $R_{14} = 120k\Omega$, $R_{15} = 60k\Omega$, $R_{16} = 1M\Omega$, $C_1 = C_2 = C_3 = 10nF$. The circuit has three integrators by using Op-amp TL082CD in a feedback loop and two multipliers IC AD633. The supplies of all active devices are ± 15 Volt. With MultiSIM 10.0, we obtain the experimental observations of system (2) as shown in Figure 5. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Sprott chaotic system is confirmed.

The hyperbolic sinusoidal function as Taylor series [35-36]:

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}. \quad (9)$$

The corresponding circuit equation of each block is given as [35]:

$$\sinh(x) = -\frac{R_{13}}{R_{16}}x - \frac{R_{13}}{R_{15}}x^3 - \frac{R_{13}}{R_{14}}x^5 \approx \delta \sinh(x), \quad (10)$$

where $\delta = -\frac{1}{100}$ is the scaling factor.

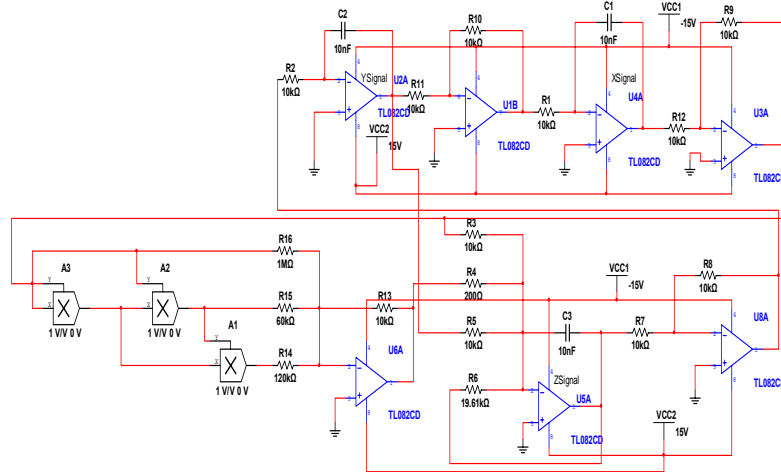
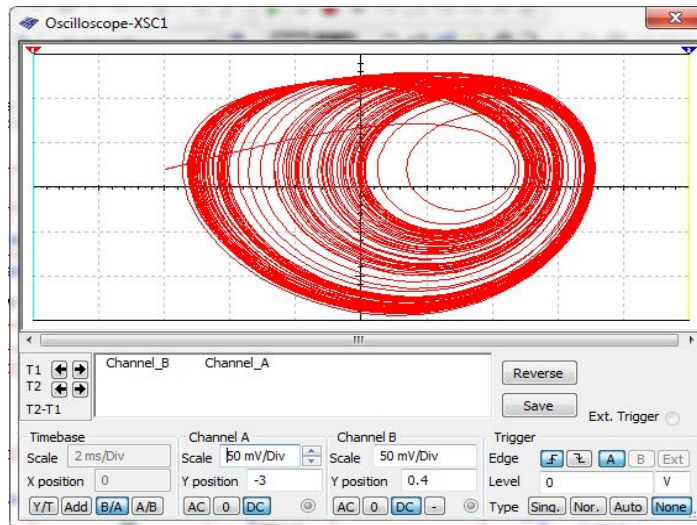
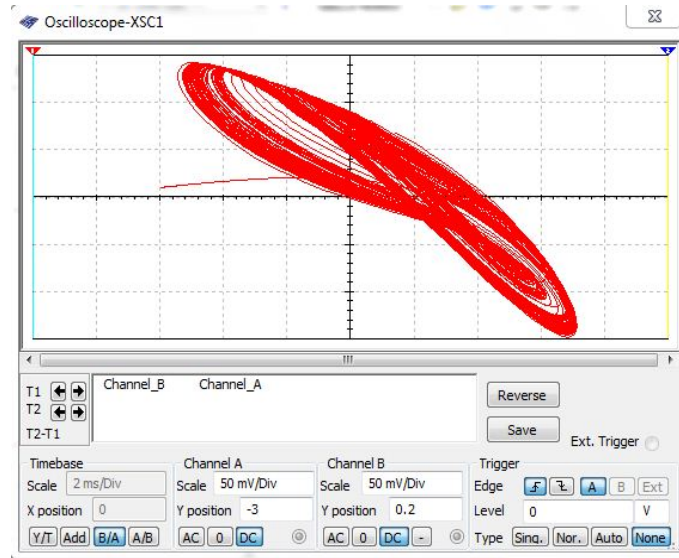


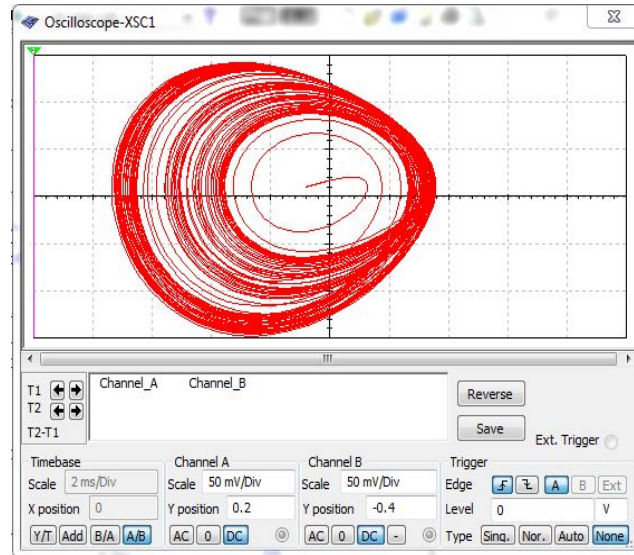
Figure 4. Schematic of the proposed Sprott chaotic system with one hyperbolic sinusoidal nonlinearity by using MultiSIM 10.0.



(a)



(b)



(c)

Figure 5. Various projections of the chaotic attractor using MultiSIM in $x - y$ plane, (b) $x - z$ plane and (c) $y - z$ plane.

Conclusion

A Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is constructed and analyzed. The fundamental properties of the system such as equilibrium points, Lyapunov exponents and Poincarémap as well as its phase portraits were described in detail. Moreover, it is implemented via a designed circuit with MultiSIM and numerical simulation using MATLAB, showing very good agreement with the simulation result. Hence, we can apply this Sprott chaotic system with one hyperbolic sinusoidal nonlinearity in practical applications like robotic, random bits generator and secure communications.

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**Numerical Simulation and Circuit Implementation for a Sprott
Chaotic System with One Hyperbolic
Sinusoidal Nonlinearity**

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Abstract

In this paper, the phenomenon of chaos that produced in the case of Sprott chaotic system with one hyperbolic sinusoidal nonlinearity, have been studied extensively. The basic dynamical properties of such system are discovered through equilibrium points, phase portrait, Lyapunov exponents and Poincaré map. Furthermore, An electronic circuit realization of the proposed system is presented in details. Finally, the circuit experimental results of the chaotic attractors show agreement with numerical simulations.

Keywords: Sprott chaotic system, Lyapunov exponents and Poincaré

Introduction

Chaos is used to explain the behavior of certain dynamical complex, i.e., systems whose state variables evolve with time, which may exhibit dynamics that are highly sensitive to initial conditions. Henri Poincaré was the first discoverer of chaos. In 1890, while studying the three-body problem, he found that there existed some orbits which are non-periodic [1-2]. Interest in nonlinear dynamics and in particular chaotic dynamics has grown rapidly since 1963, Edward Lorenz, the MIT meteorologist, he was designing a 3-D model for weather prediction [3].

In the literature, Sprott (1994) was the first to introduce a simple flow with no equilibrium points [4], In 2000, Malasoma proposed the simplest dissipative jerk equation that is parity invariant [5], Sun and Sprott (2009) constructed a piecewise exponential jerk system [6], In 2010, Sprott gives a 3-D jerk chaotic system having six terms on the R.H.S. with one hyperbolic tangential nonlinearity [7], In 2014, Chunbiao Li and Sprott constructed the chaotic flows with a single nonquadratic term [8] and in 2015, Vaidyanathan created a six-term novel Jerk chaotic system with two exponential nonlinearities [9].

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Motivated by the above researches, a Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is proposed in this work, In Section 2, we present Sprott system from three first-order autonomous ODEs, numerical results in evolving phase portraits, Lyapunov exponent's analysis and Poincaré map analysis. In section 3, basic dynamical properties of the Sprott chaotic system are discussed including equilibria and Jacobian matrices. In Section 4 we

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The Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity [28] is described by:

$$\ddot{x} + 0.6\dot{x} + \dot{x} = x - 0.5 \sinh x \quad (1)$$

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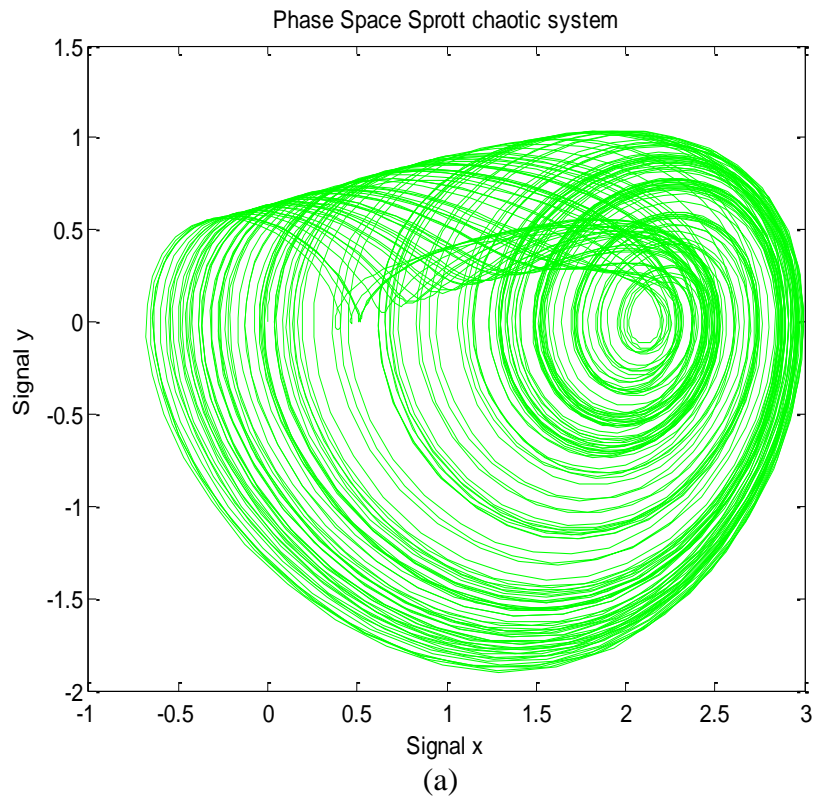
$$\left. \begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= x - a \sinh(x) - y - bz \end{aligned} \right\} \quad (2)$$

Where x, y, z are state variables and when $a = 0.5$ and $b=0.51$, the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2) exhibits strange attractor, we have chosen the initial conditions for the Sprott system (2) as

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The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity can be characterized with its Lyapunov exponents which are computed numerically by Wolf algorithm proposed in Ref. [29]. Figure 2 shows the Lyapunov exponents of the Sprott system for constant parameter $a = 0.5$ and $b=0.51$. The Sprott chaotic system is chaotic with a positive Lyapunov exponents. In addition, the Poincare map of the system in Figure 3 also reflects properties of chaos.



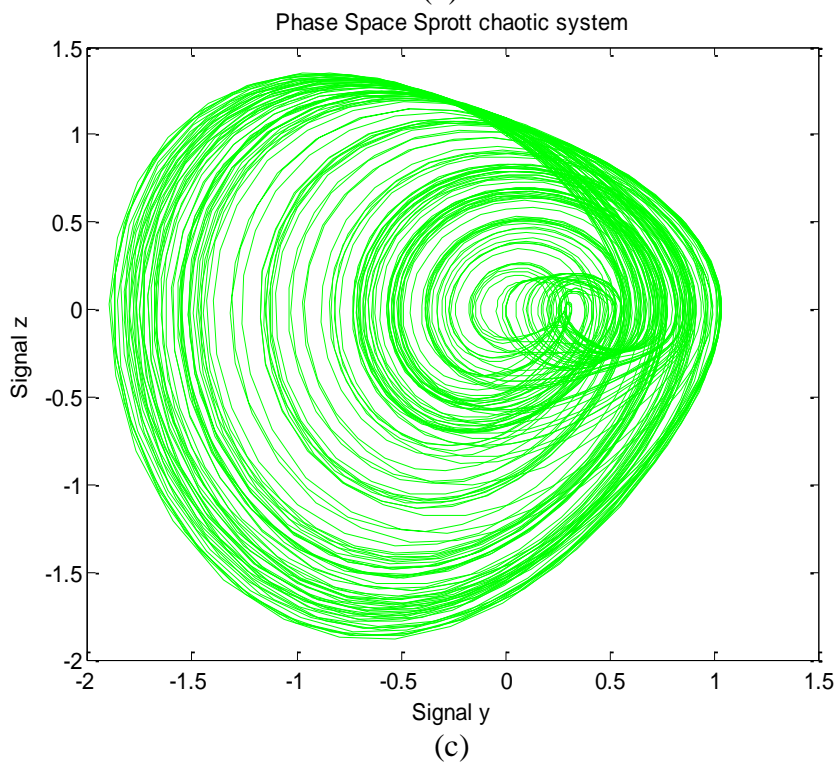
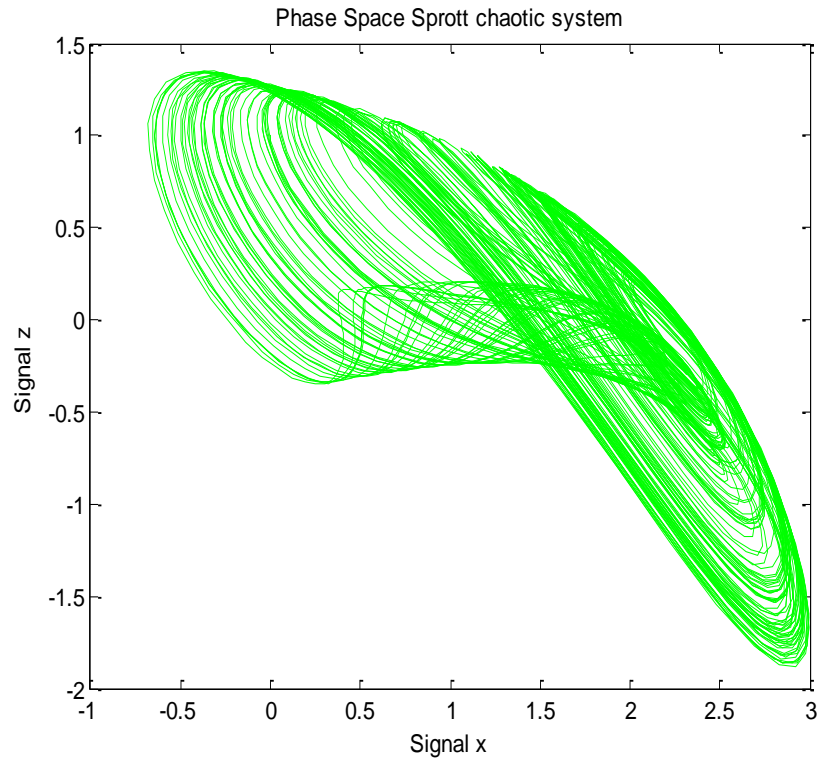


Figure 1 Numerical simulation results using MATLAB 2010, for $a = 0.5$ and $b = 0.51$, in (a) x - y plane, (b) x - z plane, (c) y - z plane.

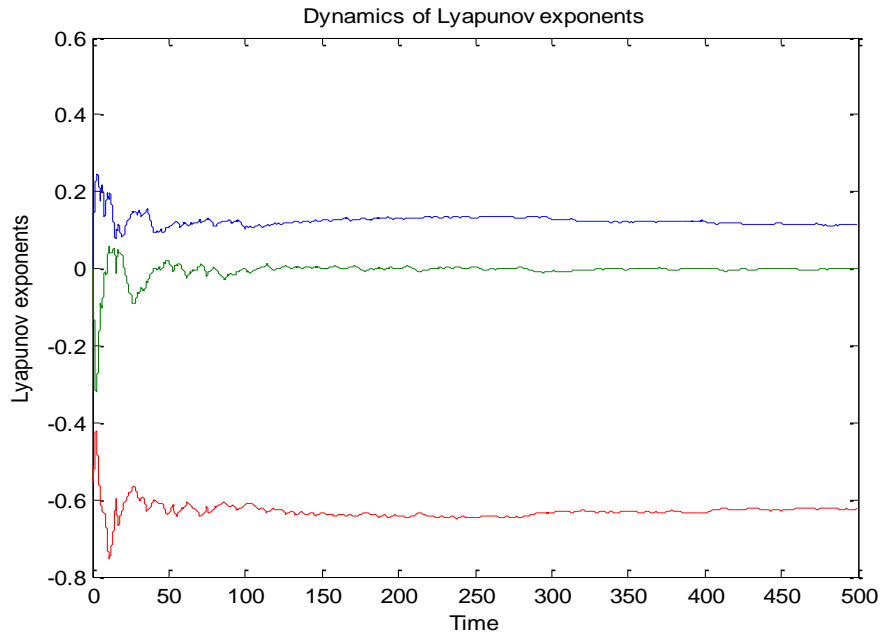


Figure 2 The dynamics of Lyapunov exponents of Sprott chaotic system for $a = 0.5$ and $b = 0.51$, using MATLAB 2010

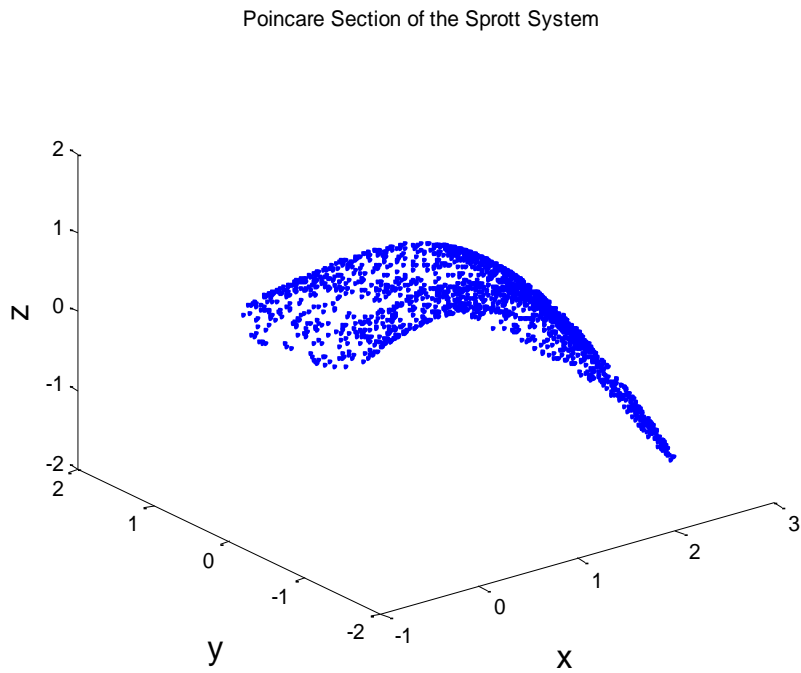


Figure 3 Poincare map in the x - y - z space plane when $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

Basic Properties of the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

The equilibrium points of (2) denote by $E(\bar{x}, \bar{y}, \bar{z})$, are the zeros of its non-linear algebraic system which can be written as:

$$\left. \begin{aligned} 0 &= y \\ 0 &= z \\ 0 &= x - a \sinh(x) - y - bz \end{aligned} \right\} \quad (4)$$

The Sprott chaotic system with one hyperbolic sinusoidal nonlinearity has one equilibrium point $E_0(0, 0, 0)$. The dynamical behavior of equilibrium point can be studied by computing the eigenvalue of the Jacobian matrix J of system (2) where:

$$J(\bar{x}, \bar{y}, \bar{z}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 - 0.5 \cosh(x) & -1 & -0.51 \end{bmatrix} \quad (5)$$

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$$J(0,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & -1 & -0.51 \end{bmatrix} \quad (6)$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.51\lambda^2 + \lambda - 0.5 \quad (7)$$

Yielding eigenvalues of $\lambda_1 = 0.3753$, $\lambda_2, \lambda_3 = -0.4426 \pm 1.0659 i$, where λ_1 is a positive real number and λ_2, λ_3 are a pair of complex conjugate eigenvalues with negative real parts, which indicates that these two imaginary equilibrium points are saddle points. This equilibrium point is unstable.

Circuit Realization of the the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

In this section, we design an electronic circuit modeling of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity. The circuit in Figure 4 has been designed following an approach based on operational amplifiers [30-34] where the state variables x, y, z of the system (2) are associated with the voltages across the capacitors C_1, C_2 and C_3 , respectively. The nonlinear term of system (2) are implemented with the analog multiplier. By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\left. \begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} y \\ \dot{y} &= \frac{1}{C_2 R_2} z \\ \dot{z} &= \frac{1}{C_3 R_3} x - \frac{1}{C_3 R_4} \sinh(x) - \frac{1}{C_3 R_5} y - \frac{1}{C_3 R_6} z \end{aligned} \right\} \quad (8)$$

We choose $R_1 = R_2 = R_3 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 10 \text{ k}\Omega$, $R_4 = 200\Omega$, $R_6 = 19.61 \text{ k}\Omega$, $R_{14} = 120 \text{ k}\Omega$, $R_{15} = 60 \text{ k}\Omega$, $R_{16} = 1 \text{ M}\Omega$, $C_1 = C_2 = C_3 = 10 \text{ nF}$. The circuit has three integrators by using Op-amp TL082CD in a feedback loop and two multipliers IC AD633. The supplies of all active devices are $\pm 15 \text{ Volt}$. With MultiSIM 10.0, we obtain the experimental observations of system (2) as shown in Figure 5. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Sprott chaotic system is confirmed.

The hyperbolic sinusoidal function as Taylor series [35-36]:

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \quad (9)$$

The corresponding circuitual equation of each block is given as [35]:

$$\sinh(x) = -\frac{R_{13}}{R_{16}}x - \frac{R_{13}}{R_{15}}x^3 - \frac{R_{13}}{R_{14}}x^5 \approx \delta \sinh(x) \quad (10)$$

where $\delta = -\frac{1}{100}$ is the scaling factor.

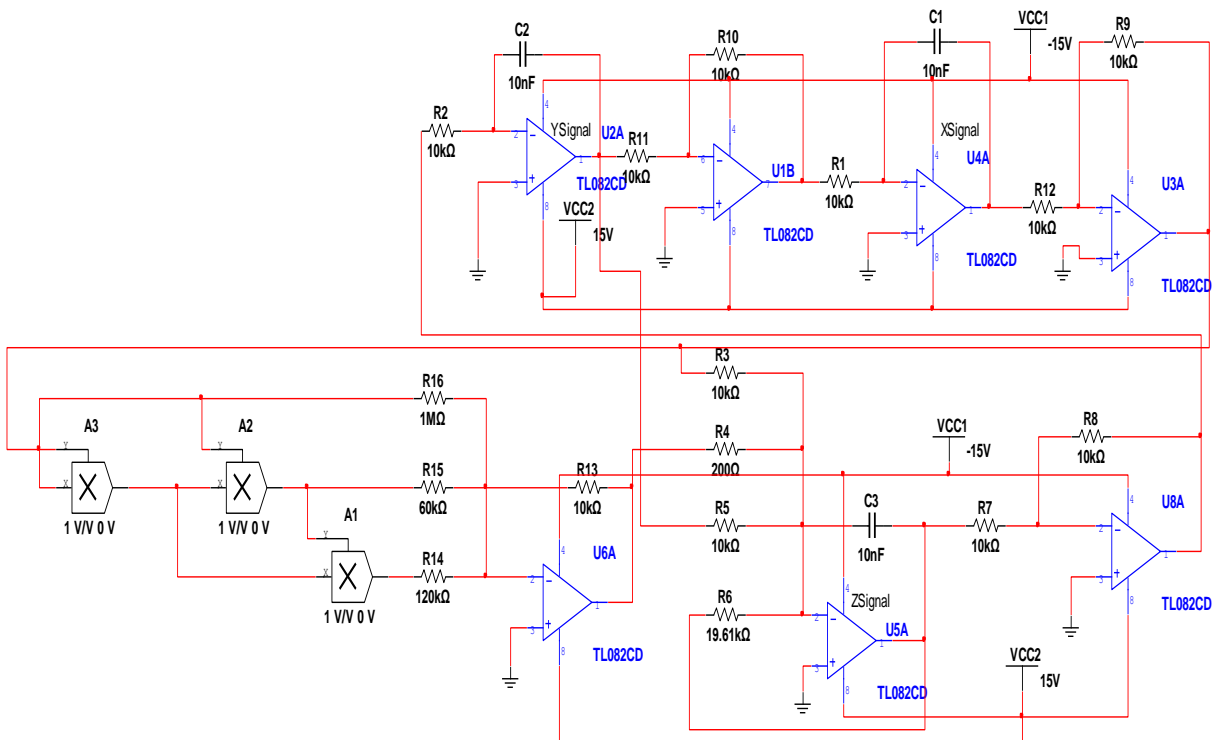
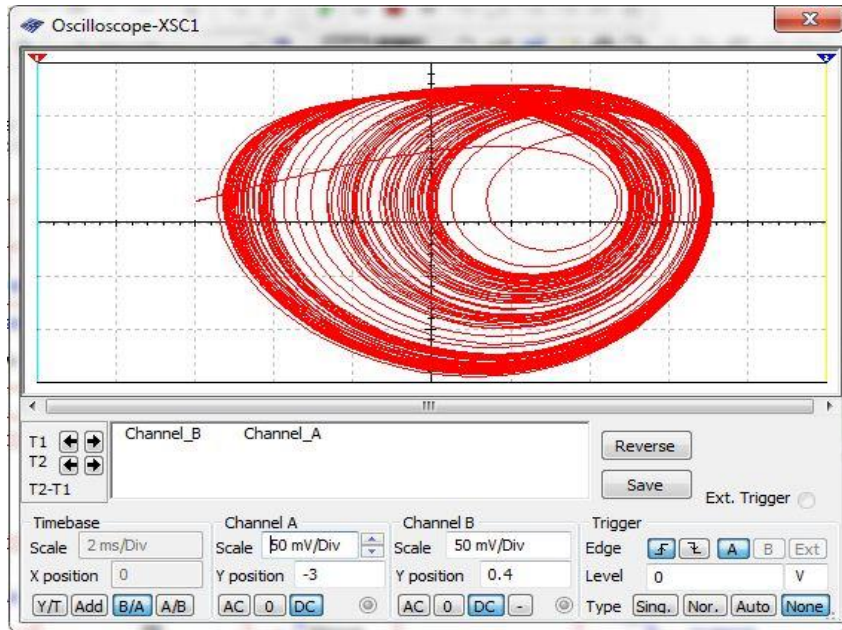
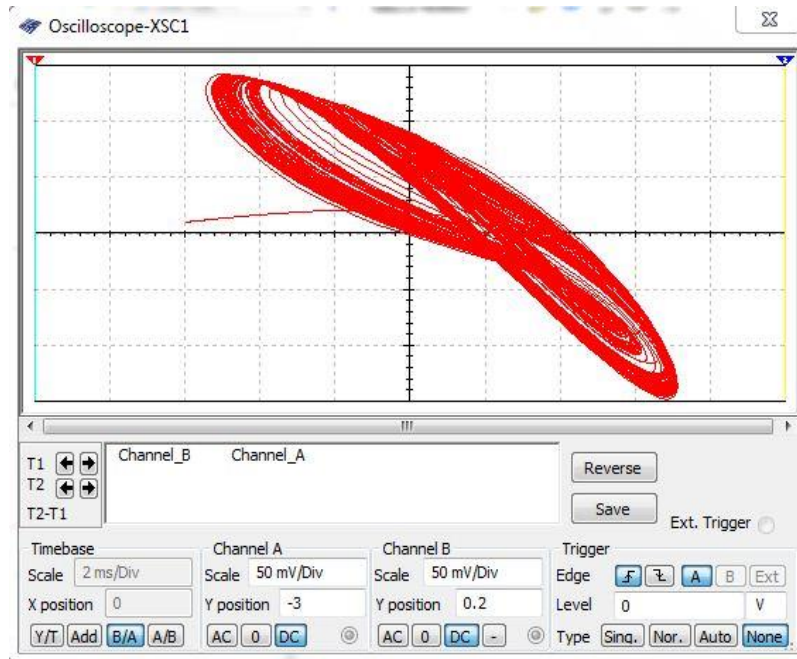


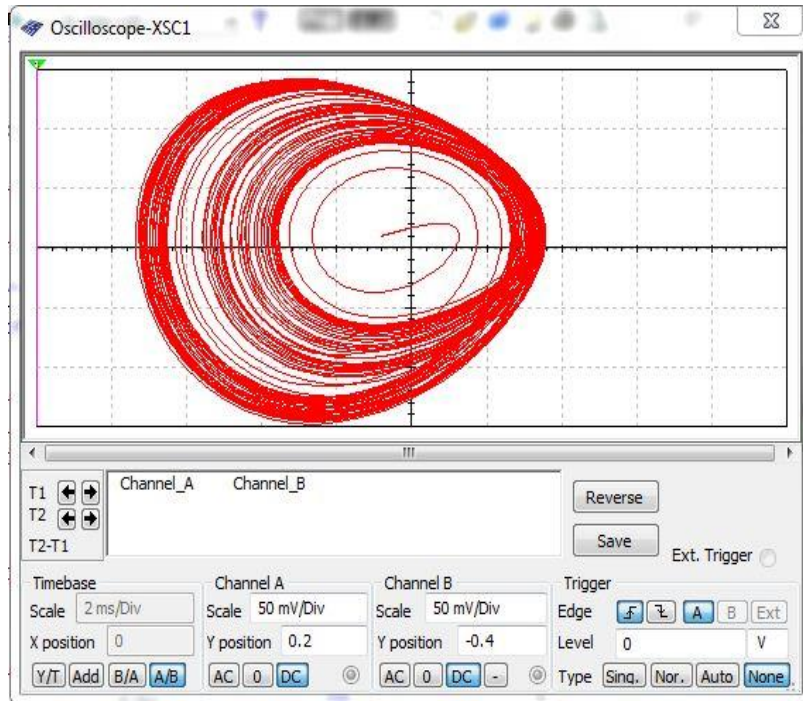
Figure 4 Schematic of the proposed Sprott chaotic system with one hyperbolic sinusoidal nonlinearity by using MultiSIM 10.0.



(a)



(b)



(c)

Figure 5 Various projections of the chaotic attractor using MultiSIM in x - y plane, (b) x - z plane and (c) y - z plane

Conclusion

A Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is constructed and analyzed. The fundamental properties of the system such as equilibrium points, Lyapunov exponents and Poincaré map as well as its phase portraits were described in detail. Moreover, it is implemented via a designed circuit with MultiSIM and numerical simulation using MATLAB, showing very good agreement with the simulation result. Hence, we can apply this Sprott chaotic system with one hyperbolic sinusoidal nonlinearity in practical applications like robotic, random bits Generator and secure communications

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To

Professor Aceng Sambas
Department of Mechanical Engineering
Universitas Muhammadiyah Tasikmalaya
Indonesia

May 4, 2017

Dear Professor Sambas,

I am happy to inform you that your revised paper entitled "*Numerical simulation and circuit implementation for a Sprott chaotic system with one hyperbolic sinusoidal nonlinearity*" jointly written with Mujiarto, Mustafa Mamat and W. S. Mada Sanjaya has been accepted for publication in the Far East Journal of Mathematical Sciences (FJMS).

Thank you for submitting your work to Far East Journal of Mathematical Sciences (FJMS).

Yours sincerely



(ARUN AZAD)



**NUMERICAL SIMULATION AND CIRCUIT
IMPLEMENTATION FOR A SPROTT CHAOTIC
SYSTEM WITH ONE HYPERBOLIC
SINUSOIDAL NONLINEARITY**

**Aceng Sambas^{a,b}, Mujiarto^a, Mustafa Mamat^{b,*} and
W. S. Mada Sanjaya^c**

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Abstract

In this paper, the phenomenon of chaos that produced in the case of Sprott chaotic system with one hyperbolic sinusoidal nonlinearity has been studied extensively. The basic dynamical properties of such system are discovered though equilibrium points, phase portrait,

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Lyapunov exponents and Poincaré map. Furthermore, an electronic circuit realization of the proposed system is presented in details. Finally, the circuit experimental results of the chaotic attractors show agreement with numerical simulations.

1. Introduction

Chaos is used to explain the behavior of certain dynamical complex, i.e., systems whose state variables evolve with time, which may exhibit dynamics that are highly sensitive to initial conditions. Henri Poincaré was the first discoverer of chaos. In 1890, while studying the three-body problem, he found that there existed some orbits which are non-periodic [1-2]. Interest in nonlinear dynamics and in particular chaotic dynamics has grown rapidly since 1963, Edward Lorenz, the MIT meteorologist, he was designing a 3-D model for weather prediction [3].

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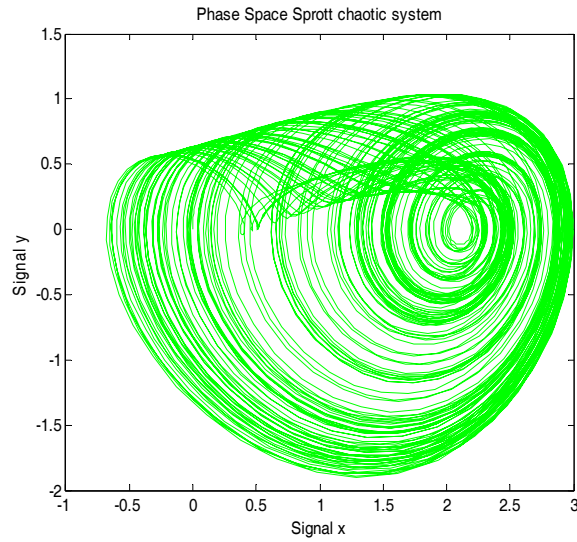
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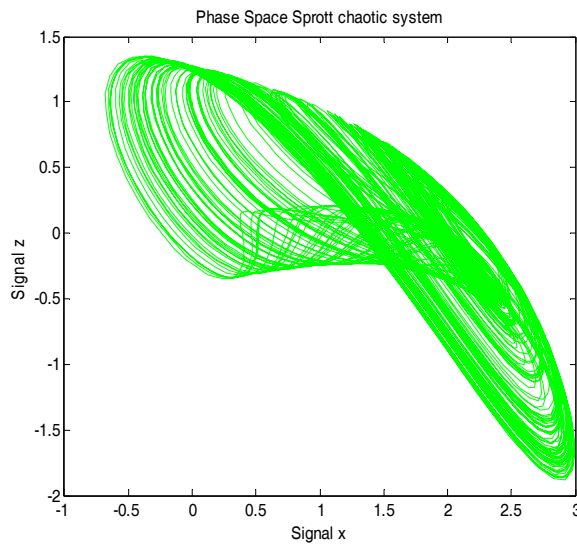
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The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity can be characterized with its Lyapunov exponents which are computed numerically by Wolf algorithm proposed in [29]. Figure

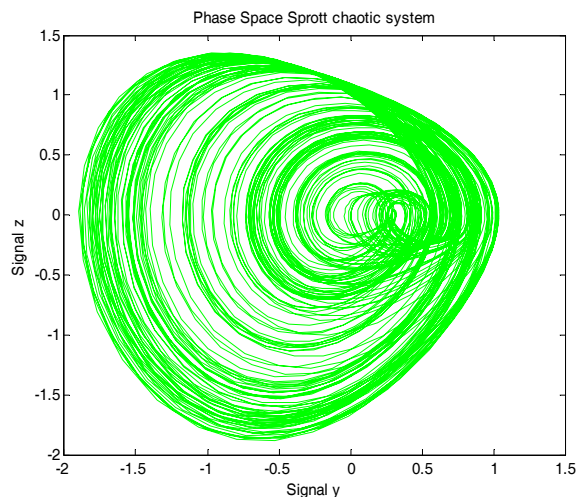
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(a)



(b)



(c)

Figure 1. Numerical simulation results using MATLAB 2010, for $a = 0.5$ and $b = 0.51$, in (a) $x - y$ plane, (b) $x - z$ plane, (c) $y - z$ plane.

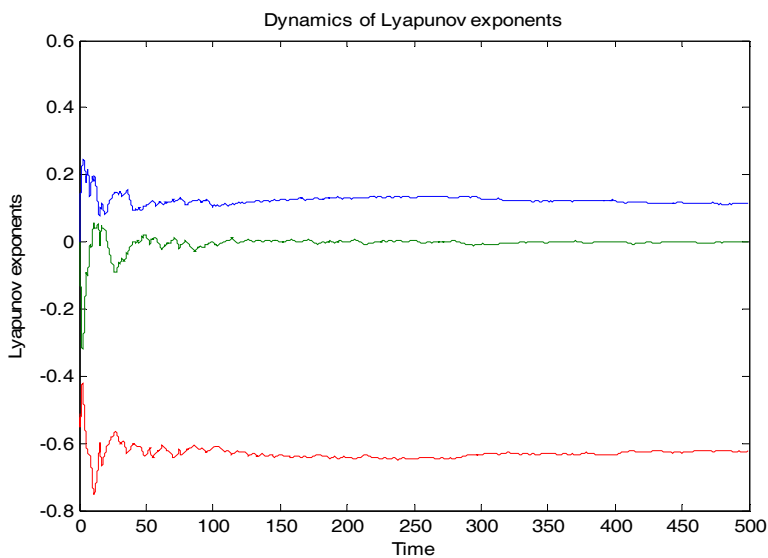


Figure 2. The dynamics of Lyapunov exponents of Sprott chaotic system for $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

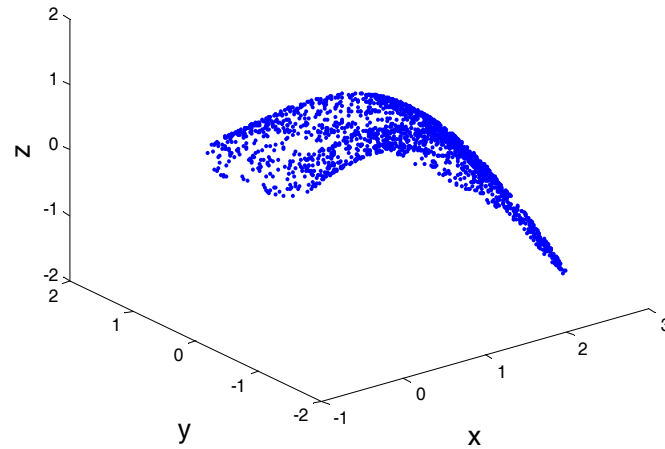


Figure 3. Poincaré map in the $x - y - z$ space plane when $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

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In this section, we design an electronic circuit modeling of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity. The circuit in Figure 4 has been designed following an approach based on operational amplifiers [30-34], where the state variables x, y, z of the system (2) are associated with the voltages across the capacitors C_1, C_2 and C_3 , respectively. The nonlinear term of system (2) are implemented with the analog multiplier. By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\left. \begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} y \\ \dot{y} &= \frac{1}{C_2 R_2} z \\ \dot{z} &= \frac{1}{C_3 R_3} x - \frac{1}{C_3 R_4} \sinh(x) - \frac{1}{C_3 R_5} y - \frac{1}{C_3 R_6} z \end{aligned} \right\}. \tag{8}$$

We choose $R_1 = R_2 = R_3 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 10k\Omega$, $R_4 = 200\Omega$, $R_6 = 19.61k\Omega$, $R_{14} = 120k\Omega$, $R_{15} = 60k\Omega$, $R_{16} = 1M\Omega$, $C_1 = C_2 = C_3 = 10nF$. The circuit has three integrators by using Op-amp TL082CD in a feedback loop and two multipliers IC AD633. The supplies of all active devices are ± 15 Volt. With MultiSIM 10.0, we obtain the experimental observations of system (2) as shown in Figure 5. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Sprott chaotic system is confirmed.

The hyperbolic sinusoidal function as Taylor series [35, 36]:

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}. \tag{9}$$

The corresponding circuital equation of each block is given as [35]:

$$\sinh(x) = -\frac{R_{13}}{R_{16}}x - \frac{R_{13}}{R_{15}}x^3 - \frac{R_{13}}{R_{14}}x^5 \approx \delta \sinh(x), \tag{10}$$

where $\delta = -\frac{1}{100}$ is the scaling factor.

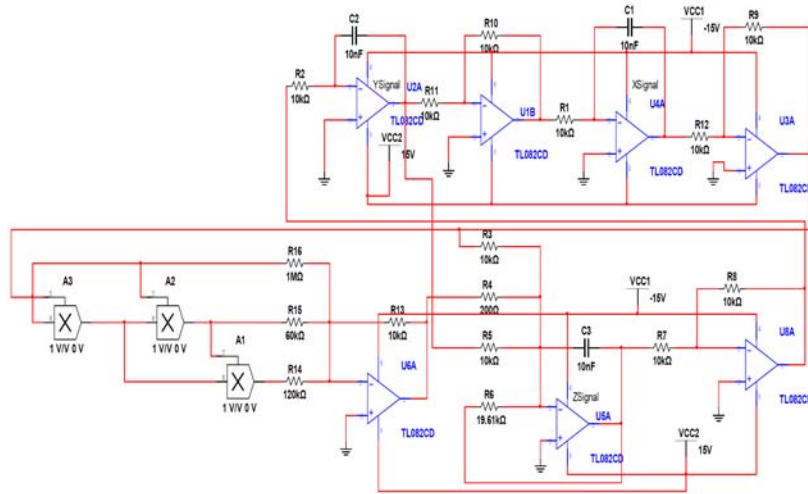
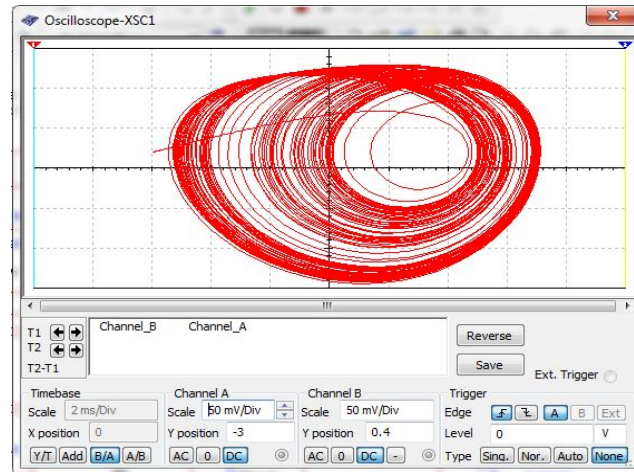
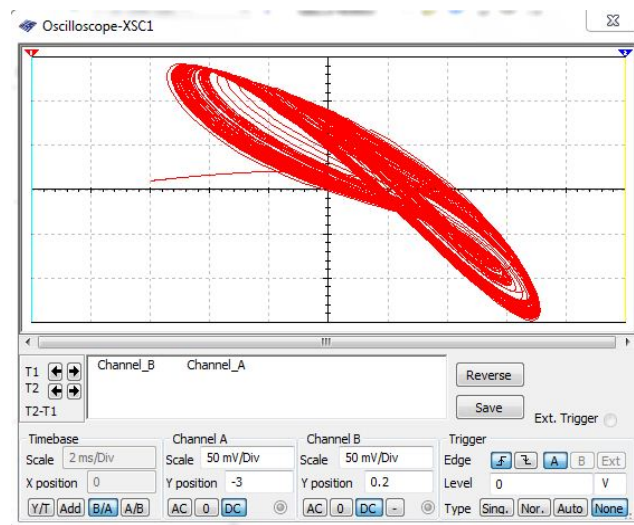


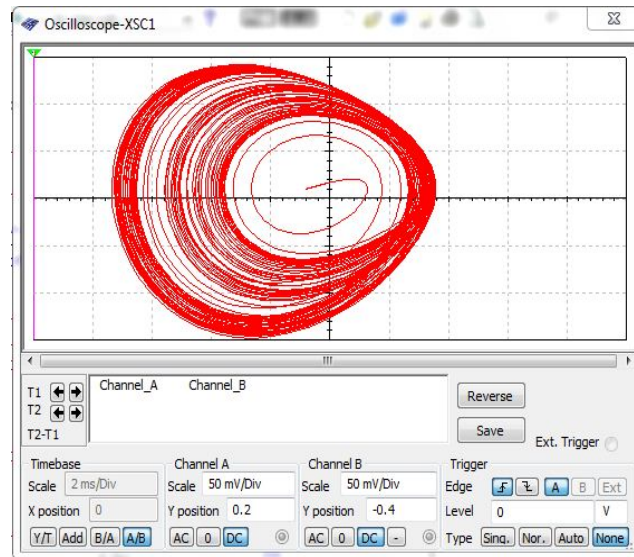
Figure 4. Schematic of the proposed Sprott chaotic system with one hyperbolic sinusoidal nonlinearity by using MultiSIM 10.0.



(a)



(b)



(c)

Figure 5. Various projections of the chaotic attractor using MultiSIM in $x - y$ plane, (b) $x - z$ plane and (c) $y - z$ plane.

5. Conclusion

A Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is constructed and analyzed. The fundamental properties of the system such as equilibrium points, Lyapunov exponents and Poincaré map as well as its phase portraits were described in detail. Moreover, it is implemented via a designed circuit with MultiSIM and numerical simulation using MATLAB, showing very good agreement with the simulation result. Hence, we can apply this Sprott chaotic system with one hyperbolic sinusoidal nonlinearity in practical applications like robotic, random bits generator and secure communications.

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