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NUMERICAL SIMULATION AND CIRCUIT IMPLEMENTATION FOR A SPROTT CHAOTIC SYSTEM WITH ONE HYPERBOLIC SINUSOIDAL NONLINEARITY

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Abstract

In this paper, the phenomenon of chaos that produced in the case of Sprott chaotic system with one hyperbolic sinusoidal nonlinearity has been studied extensively. The basic dynamical properties of such system are discovered though equilibrium points, phase portrait,

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Lyapunov exponents and Poincaré map. Furthermore, an electronic circuit realization of the proposed system is presented in details. Finally, the circuit experimental results of the chaotic attractors show agreement with numerical simulations.

1. Introduction

Chaos is used to explain the behavior of certain dynamical complex, i.e., systems whose state variables evolve with time, which may exhibit dynamics that are highly sensitive to initial conditions. Henri Poincaré was the first discoverer of chaos. In 1890, while studying the three-body problem, he found that there existed some orbits which are non-periodic [1-2]. Interest in nonlinear dynamics and in particular chaotic dynamics has grown rapidly since 1963, Edward Lorenz, the MIT meteorologist, he was designing a 3-D model for weather prediction [3].

In the literature, Sprott [4] was the first to introduce a simple flow with no equilibrium points. In [5], Malasoma proposed the simplest dissipative jerk equation that is parity invariant, Sun and Sprott [6] constructed a piecewise exponential jerk system. In [7], Sprott gave a 3-D jerk chaotic system having six terms on the R.H.S. with one hyperbolic tangential nonlinearity. In [8], Li and Sprott constructed the chaotic flows with a single nonquadratic term and in [9], Vaidyanathan et al. created a six-term novel jerk chaotic system with two exponential nonlinearities.

Chaos has been widely applied to many scientific disciplines, such as ecology [10], biology [11], economy [12], random bit generators [13], psychology [14], laser [15], astronomy [16], chemical reaction [17], robotics [18], text encryption [19], image encryption [20], voice encryption [21], secure communication systems [22-27] etc.

Motivated by the above researches, a Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is proposed in this work. In Section 2, we present Sprott system from three first-order autonomous ODEs, numerical results in evolving phase portraits, Lyapunov exponent’s analysis and Poincaré map analysis. In Section 3, basic dynamical properties of the Sprott chaotic system are discussed including equilibria and Jacobian matrices. In Section 4 we present an electronic circuit that implements the nonlinear system and Finally, Section 5 contains the conclusion remarks.

2. The Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity [28] is described by:

$$\dddot{x} + 0.6\ddot{x} + \dot{x} = x - 0.5\sinh x. \tag{1}$$

In system form, the differential equation (1) can be expressed as:

$$\left.\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= x - a\sinh(x) - y - bz \end{aligned}\right\}, \tag{2}$$

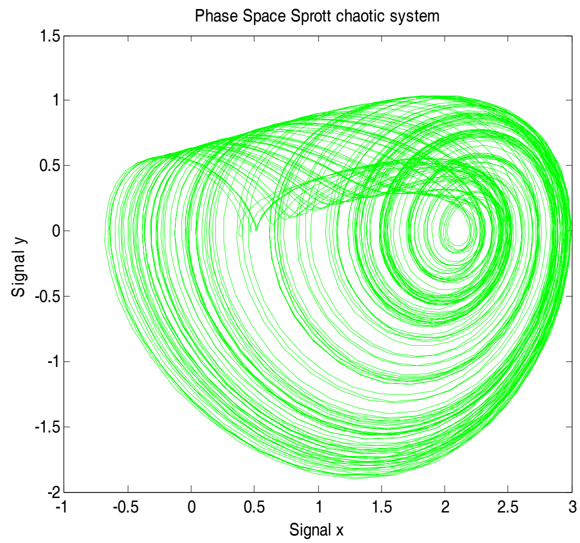
where x, y, z are state variables and when $a = 0.5$ and $b = 0.51$, the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2) exhibits strange attractor, we have chosen the initial conditions for the Sprott system (2) as

$$x_0 = 0,\ y_0 = 0,\ z_0 = 1. \tag{3}$$

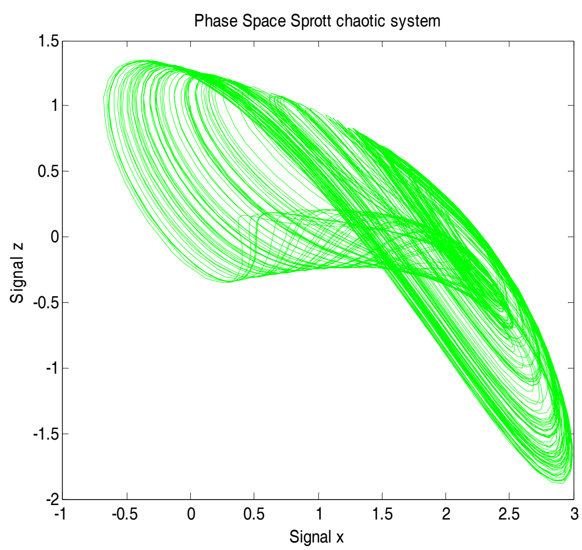
For numerical simulation of chaotic system defined by a set of differential equation such as the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2), different integration techniques can be used. In the MATLAB 2010, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used. Figures 1(a)-(c) show the projections of the phase space orbit on to the $x - y$ plane, the $x - z$ plane and the $y - z$ plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity (2) presents chaotic attractors of jerk attractor type.

The dynamics of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity can be characterized with its Lyapunov exponents which are computed numerically by Wolf algorithm proposed in [29]. Figure

2 shows the Lyapunov exponents of the Sprott system for constant parameter $a = 0.5$ and $b = 0.51$. The Sprott chaotic system is chaotic with positive Lyapunov exponents. In addition, the Poincaré map of the system in Figure 3 also reflects properties of chaos.



(a)



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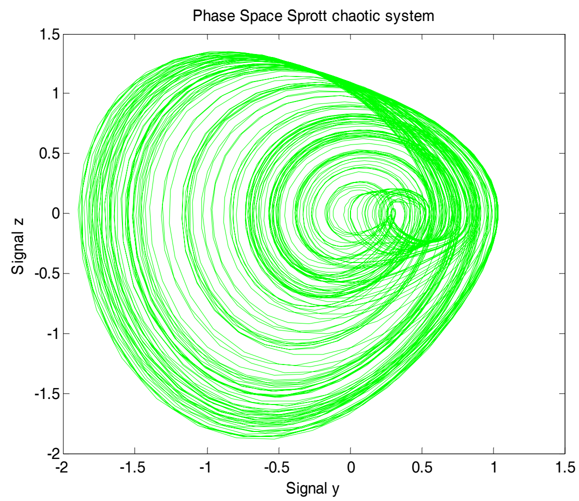


(c)

Figure 1. Numerical simulation results using MATLAB 2010, for $a = 0.5$ and $b = 0.51$, in (a) $x - y$ plane, (b) $x - z$ plane, (c) $y - z$ plane.

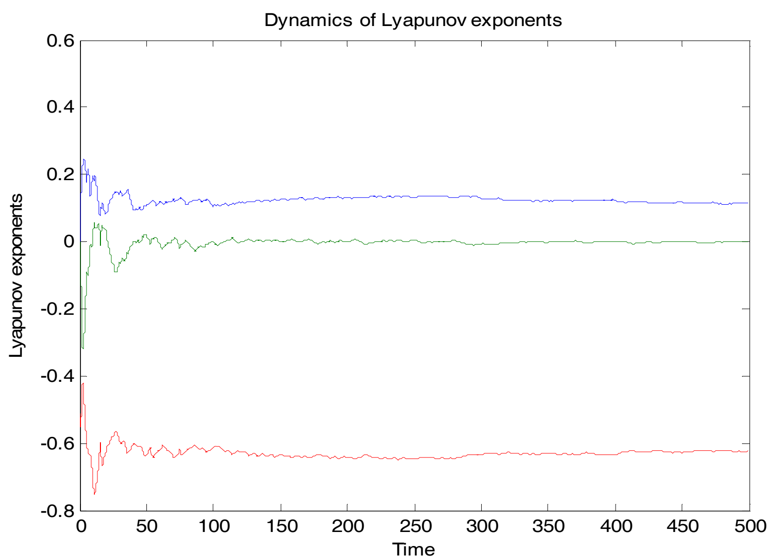


Figure 2. The dynamics of Lyapunov exponents of Sprott chaotic system for $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

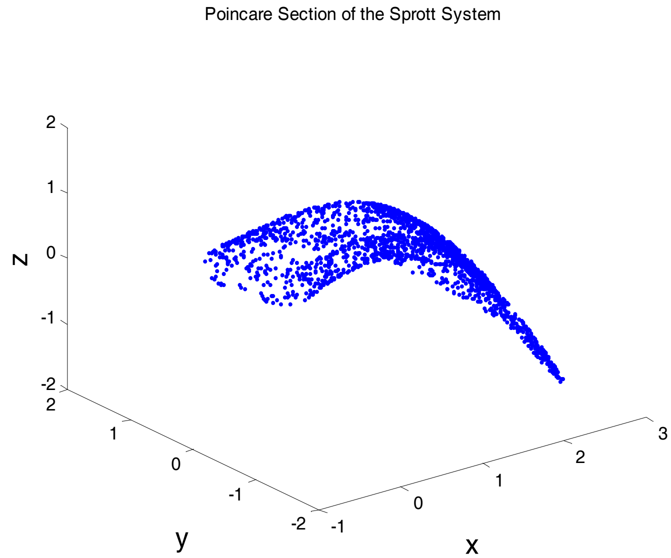


Figure 3. Poincaré map in the $x - y - z$ space plane when $a = 0.5$ and $b = 0.51$, using MATLAB 2010.

3. Basic Properties of the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

The equilibrium points of (2) denote by $E(\bar{x}, \bar{y}, \bar{z})$, are the zeros of its nonlinear algebraic system which can be written as:

$$\left.\begin{aligned} 0 &= y \\ 0 &= z \\ 0 &= x - a\sinh(x) - y - bz \end{aligned}\right\}. \quad (4)$$

The Sprott chaotic system with one hyperbolic sinusoidal nonlinearity has one equilibrium point $E_0(0, 0, 0)$. The dynamical behavior of equilibrium point can be studied by computing the eigenvalue of the Jacobian matrix J of system (2), where:

$$J(\bar{x}, \bar{y}, \bar{z}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 - 0.5\cosh(x) & -1 & -0.51 \end{bmatrix}. \quad (5)$$

For equilibrium points $E_0(0,\ 0,\ 0)$, the Jacobian becomes:

$$J(0,\ 0,\ 0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & -1 & -0.51 \end{bmatrix}. \tag{6}$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.51\lambda^2 + \lambda - 0.5. \tag{7}$$

Yielding eigenvalues of $\lambda_1 = 0.3753$, $\lambda_2, \lambda_3 = -0.4426 \pm 1.0659\, i$, where λ_1 is a positive real number and λ_2, λ_3 are a pair of complex conjugate eigenvalues with negative real parts, which indicates that these two imaginary equilibrium points are saddle points. This equilibrium point is unstable.

4. Circuit Realization of the Sprott Chaotic System with One Hyperbolic Sinusoidal Nonlinearity

In this section, we design an electronic circuit modeling of the Sprott chaotic system with one hyperbolic sinusoidal nonlinearity. The circuit in Figure 4 has been designed following an approach based on operational amplifiers [30-34], where the state variables *x*, *y*, *z* of the system (2) are associated with the voltages across the capacitors C_1, C_2 and C_3, respectively. The nonlinear term of system (2) are implemented with the analog multiplier. By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\left.\begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} y \\ \dot{y} &= \frac{1}{C_2 R_2} z \\ \dot{z} &= \frac{1}{C_3 R_3} x - \frac{1}{C_3 R_4} \sinh(x) - \frac{1}{C_3 R_5} y - \frac{1}{C_3 R_6} z \end{aligned}\right\}. \tag{8}$$

We choose $R_1 = R_2 = R_3 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 10\text{k}\Omega$, $R_4 = 200\Omega$, $R_6 = 19.61\text{k}\Omega$, $R_{14} = 120k\Omega$, $R_{15} = 60\text{k}\Omega$, $R_{16} = 1\text{M}\Omega$, $C_1 = C_2 = C_3 = 10nF$. The circuit has three integrators by using Op-amp TL082CD in a feedback loop and two multipliers IC AD633. The supplies of all active devices are ±15 Volt. With MultiSIM 10.0, we obtain the experimental observations of system (2) as shown in Figure 5. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Sprott chaotic system is confirmed.

The hyperbolic sinusoidal function as Taylor series [35, 36]:

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}. \tag{9}$$

The corresponding circuital equation of each block is given as [35]:

$$\sinh(x) = -\frac{R_{13}}{R_{16}} x - \frac{R_{13}}{R_{15}} x^3 - \frac{R_{13}}{R_{14}} x^5 \approx \delta \sinh(x), \tag{10}$$

where $\delta = -\frac{1}{100}$ is the scaling factor.

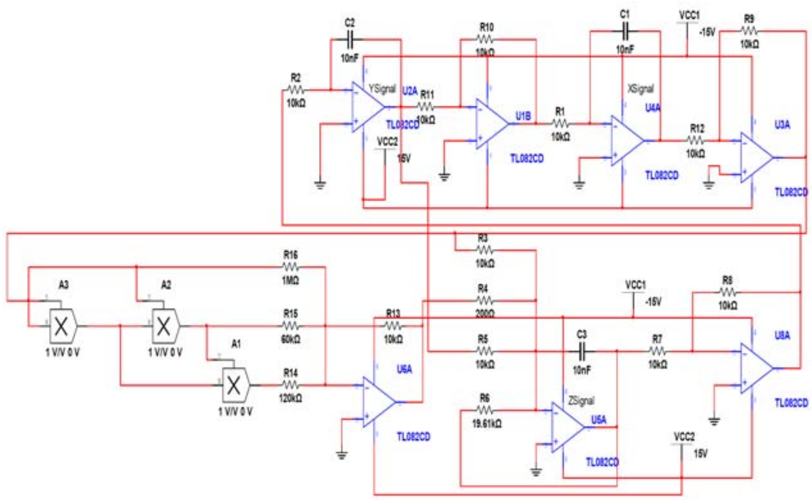
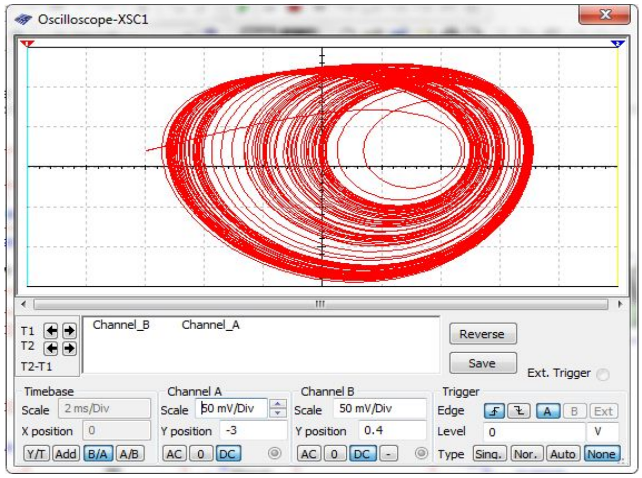


Figure 4. Schematic of the proposed Sprott chaotic system with one hyperbolic sinusoidal nonlinearity by using MultiSIM 10.0.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T1
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Y/T Add B/A A/B
Channel A
Scale 50 mV/Div
Y position -3
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Channel B
Scale 50 mV/Div
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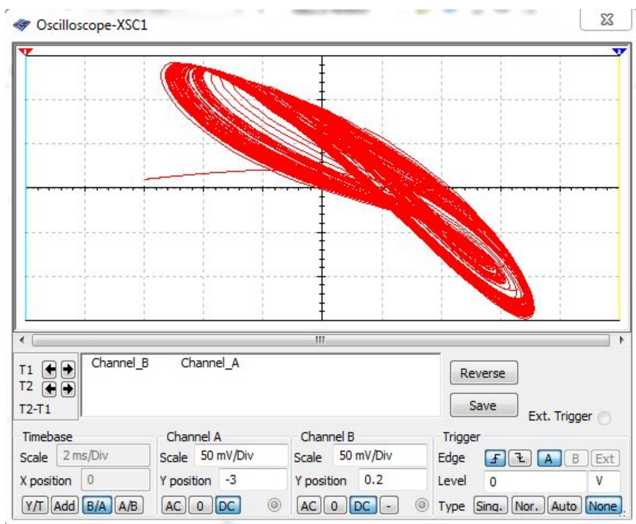
(a)


Oscilloscope-XSC1
T1
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Timebase
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X position 0
Y/T Add B/A A/B
Channel A
Scale 50 mV/Div
Y position -3
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Scale 50 mV/Div
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Trigger
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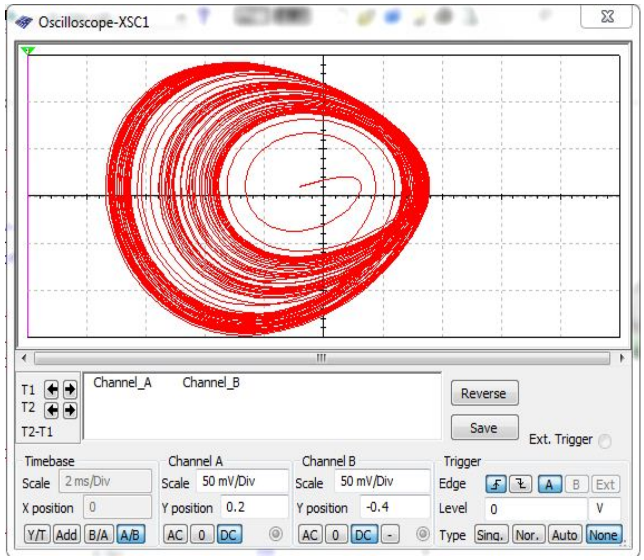
(b)



(c)

Figure 5. Various projections of the chaotic attractor using MultiSIM in $x-y$ plane, (b) $x-z$ plane and (c) $y-z$ plane.

5. Conclusion

A Sprott chaotic system with one hyperbolic sinusoidal nonlinearity is constructed and analyzed. The fundamental properties of the system such as equilibrium points, Lyapunov exponents and Poincaré map as well as its phase portraits were described in detail. Moreover, it is implemented via a designed circuit with MultiSIM and numerical simulation using MATLAB, showing very good agreement with the simulation result. Hence, we can apply this Sprott chaotic system with one hyperbolic sinusoidal nonlinearity in practical applications like robotic, random bits generator and secure communications.

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