A Novel Chaotic Hidden Attractor, its Synchronization and Circuit Implementation

*ACENG SAMBAS[1,2], MUSTAFA MAMAT[2], SUNDARAPANDIAN VAIDYANATHAN[3], MOHAMAD AFENDEE MOHAMED[2], W. S. MADA SANJAYA[4], MUJIARTO[1]

[1]Department of Mechanical Engineering,
Universitas Muhammadiyah Tasikmalaya,
INDONESIA.
[2]Faculty of Informatics and Computing,
Universiti Sultan Zainal Abidin, Kuala Terengganu,
MALAYSIA.
[3]Research and Development Centre,
Vel Tech University, Avadi, Chennai,
INDIA
[4]Department of Physics,
Universitas Islam Negeri Sunan GunungDjati,
Indonesia

*acengs@umtas.ac.id, must@unisza.edu.my, sundarvtu@gmail.com,
mafendee@unisza.edu.my, madasws@gmail.com, mujiarto@umtas.ac.id

Abstract: - A 3-D novel hidden chaotic attractor with no equilibrium point is proposed in this paper. The dynamical properties of the new chaotic system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. As an engineering application, adaptive synchronization of identical hidden chaotic attractors with no equilibrium point is designed via nonlinear control and Lyapunov stability theory. Furthermore, an electronic circuit realization of the novel hidden chaotic attractor is presented in detail to confirm the feasibility of the theoretical hidden chaotic attractor model. The outputs show that results of the system modelled in MATLAB simulation confirm the MultiSIM results.

Key-Words: - Chaos, hidden attractor, synchronization, numerical simulation, circuit design.

1 Introduction

A chaotic system is commonly defined as a nonlinear dynamical system that is highly sensitive to even small perturbations in its initial conditions [1-2].

Chaos theory has several applications in science and engineering such as oscillators [3-5], chemical reactors [6-7], biology [8], ecology [9], economic [10], robotics [11-12], magnetic bearing [13], satellite communication [14], memristors [15-16], voice encryption [17], secure communication systems [18-20].

Recently, there has been some good interest in finding and studying of chaotic systems with infinite number of equilibria such as equilibria located on the circle [21], square [22], ellipse [23], rounded square [24], line [25], heart shape [26], conic-shaped [27] and three-leaved clover [28]. In addition, 3D chaotic systems with no equilibria are reported [29-31].

There are two types of attractors: *self-excited* and *hidden* attractors. The hidden attractor is periodic or chaotic attractor in the system without equilibria or with only stable equilibrium, a special case of multi-stability [32] and coexistence of attractors [33]. Hidden attractors are important in engineering applications because it can explain perturbations in a structure like a bridge or aircraft wing, convective fluid motion in rotating cavity [34] and model of drilling system actuated by induction motor [35].

Motivated by the research on chaotic systems with hidden chaotic attractors, we prove a new 3-D novel chaotic system with hidden attractor in this paper. Section 2 describes the new chaotic system with hidden attractor and details the dynamical properties such as Lyapunov exponents and Kaplan-Yorke dimension. Section 3 describes the adaptive synchronization of the new chaotic system with unknown parameters. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 4. The circuit experimental results of the new hidden chaotic attractor show agreement with the numerical simulations. Section 5 contains the conclusions of this work.

2. A New Chaotic System

In this paper, we announce a new 3-D chaotic system given by the dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_1x_3 - bx_2 - x_2x_3 \\ \dot{x}_3 = x_2^2 - 1 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants.

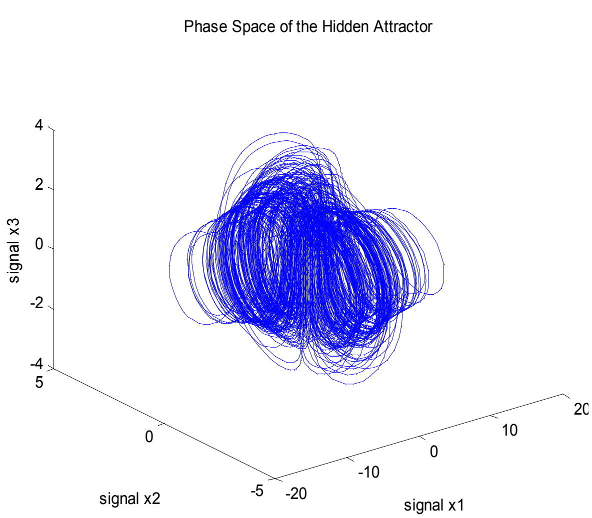
In this paper, we show that the system (1) is *chaotic* for the parameter values

$$a = 0.1, \quad b = 0.1 \tag{2}$$

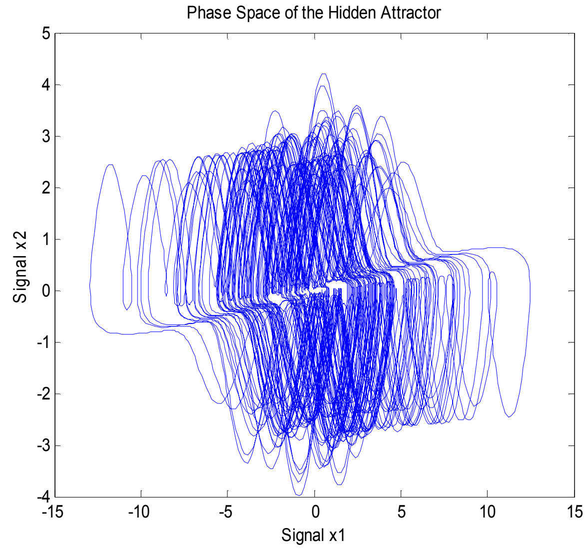
For numerical simulations, we take the initial values of the system (1) as

$$x_1(0) = 0.3, \quad x_2(0) = 0.3, \quad x_3(0) = 0.3 \tag{3}$$

Fig. 1 shows the phase portraits strange attractor of the new chaotic system (1) for the parameter values (2) and initial conditions (3). Figure 1 (a) shows the 3-D phase portrait of the new chaotic system (1). Figs. 1 (b)-(c) show the projections of the new chaotic system (1) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes, respectively



(a)



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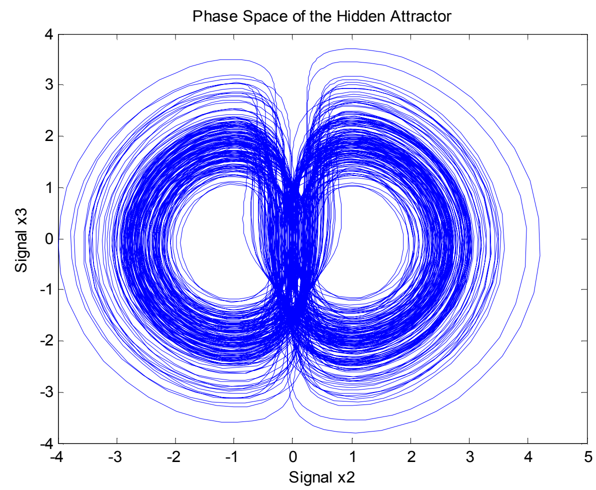
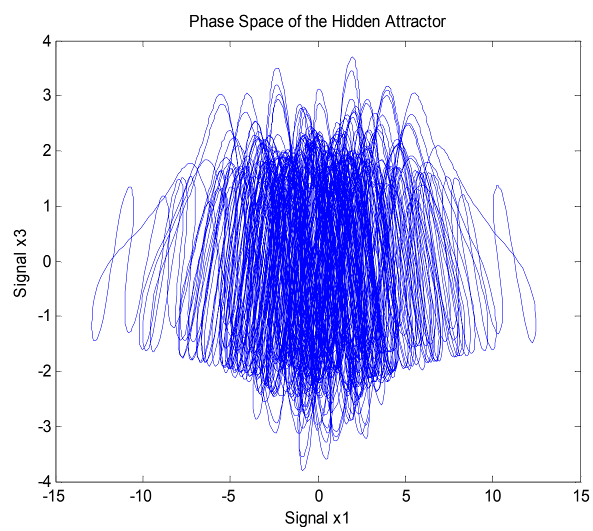


(c)



(d)

Fig 1. Phase portraits of the new chaotic system (1) for $a = 0.1, \ b = 0.1$

For the rest of this section, we take the parameter values as in the chaotic case (2), *i.e.* $a = 0.1$ and $b = 0.1$.

The equilibrium points of the new chaotic system (1) are obtained by solving the system of equations

$$x_2 = 0 \tag{4a}$$

$$ax_1x_3 - bx_2 - x_2x_3 = 0 \tag{4b}$$

$$x_2^2 - 1 = 0 \tag{4c}$$

Since (4a) and (4b) contradict each other, the new chaotic system (1) does not have any equilibrium point. This shows that the new chaotic system (1) exhibits *hidden attractor*.

We note that the new chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \tag{5}$$

for all values of the parameters. This shows that the new chaotic system (1) has a rotation symmetry about the x_3 − axis.

For the parameter values as in the chaotic case (2) and the initial state as in (3), the Lyapunov exponents of the new 3-D system (2) are determined using Wolf's algorithm as

$$L_1 = 0.0672, \quad L_2 = 0, \quad L_3 = -0.1584 \tag{6}$$

Since $L_1 > 0$, the new 3-D system (1) is chaotic. Thus, the system (1) exhibits a chaotic hidden attractor. Also, we note that the sum of the Lyapunov exponents in (6) is negative. This shows that the new 3-D chaotic system (1) is dissipative.

The Kaplan-Yorke dimension of the new 3-D system (1) is determined as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.4242, \tag{7}$$

which indicates the high complexity of the new chaotic system (1).

Since $L_1 > 0$, the new 3-D system (1) is chaotic. Thus, the system (1) exhibits a chaotic hidden attractor. Also, we note that the sum of the

Fig. 2 shows the Lyapunov exponents of the new chaotic system (1) with hidden chaotic attractor.

3. Adaptive Synchronization of the New Chaotic Systems with Hidden Attractors

In this section, we devise adaptive controller so as to synchronize the respective states of identical new chaotic systems with unknown parameters considered as *master* and *slave* systems respectively.

As the master system, we consider the new chaotic system given by

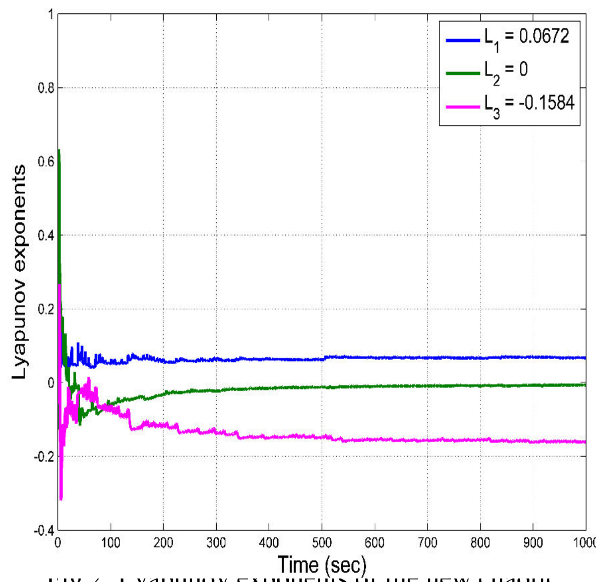


Fig 2. Lyapunov exponents of the new chaotic system (1) for $a = 0.1, \ b = 0.1$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_1x_3 - bx_2 - x_2x_3 \\ \dot{x}_3 = x_2^2 - 1 \end{cases} \tag{8}$$

where x_1, x_2, x_3 are the states and a, b are unknown parameters.

As the slave system, we consider the new chaotic system given by

$$\begin{cases} \dot{y}_1 = y_2 + u_1 \\ \dot{y}_2 = ay_1y_3 - by_2 - y_2y_3 + u_2 \\ \dot{y}_3 = y_2^2 - 1 + u_3 \end{cases} \tag{9}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be designed.

The synchronization error between the systems (8) and (9) is defined as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{10}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_2 + u_1 \\ \dot{e}_2 = a(y_1y_3 - x_1x_3) - be_2 - y_2y_3 + x_2x_3 + u_2 \\ \dot{e}_3 = y_2^2 - x_2^2 + u_3 \end{cases} \tag{11}$$

We consider the adaptive control defined by

$$\begin{cases} u_1 = -e_2 - k_1 e_1 \\ u_2 = -\hat{a}(t)(y_1 y_3 - x_1 x_3) + \hat{b}(t) e_2 + y_2 y_3 - x_2 x_3 - k_2 e_2 \\ u_3 = -y_2^2 + x_2^2 - k_3 e_3 \end{cases} \quad (12)$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (12) into (11), we obtain the closed-loop system

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = [a - \hat{a}(t)](y_1 y_3 - x_1 x_3) - [b - \hat{b}(t)] e_2 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \quad (13)$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \quad (14)$$

Using (14), we can simplify (13) as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = e_a (y_1 y_3 - x_1 x_3) - e_b e_2 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \quad (15)$$

Differentiating (14) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \quad (16)$$

Next, we consider the Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right) \quad (17)$$

which is positive definite on R^5.

Differentiating V along the trajectories of (15) and (16), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_2 (y_1 y_3 - x_1 x_3) - \dot{\hat{a}}\right] + e_b\left(-e_2^2 - \dot{\hat{b}}\right) \quad (18)$$

In view of (18), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}} = e_2 (y_1 y_3 - x_1 x_3) \\ \dot{\hat{b}} = -e_2^2 \end{cases} \quad (19)$$

Next, we prove the main theorem of this section.

Theorem 2. The new chaotic systems (8) and (9) with unknown parameters are globally and asymptotically stabilized by the adaptive control law (12) and the parameter update law (19), where k_1, k_2, k_3 are positive constants.

Proof. The Lyapunov function V defined by (17) is quadratic and positive definite on R^5.

By substituting the parameter update law (19) into (18), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (20)$$

which is negative semi-definite on R^5.

Thus, by Barbalat's lemma [36], it follows that the closed-loop system (15) is globally asymptotically stable for all initial conditions $e(0) \in R^3$.

Hence, we conclude that the new chaotic systems (8) and (9) with unknown parameters are globally and asymptotically stabilized by the adaptive control law (12) and the parameter update law (19), where k_1, k_2, k_3 are positive constants.

This completes the proof. ■

For numerical simulations, we take the gain constants as

$$k_1 = 10, \quad k_2 = 10, \quad k_3 = 10 \quad (21)$$

We take the parameter values as in the chaotic case (2), i.e.

$$a = 0.1, \quad b = 0.1 \quad (22)$$

We take the initial conditions of the states of the master system (8) as

$$x_1(0) = 6.4, \quad x_2(0) = 12.3, \quad x_3(0) = 4.7 \quad (23)$$

We take the initial conditions of the states of the slave system (9) as

$$y_1(0) = 10.2, \quad y_2(0) = 7.4, \quad y_3(0) = 12.5 \quad (24)$$

We take the initial conditions of the parameter estimates as

$$\hat{a}(0) = 5.3, \quad \hat{b}(0) = 11.1 \quad (25)$$

Fig. 3 shows the synchronization of the states of the new chaotic systems (8) and (9).

Fig. 4 shows the time-history of the synchronization errors e_1, e_2, e_3.

4. Circuit Implementation of the New Chaotic System

An electronic circuit which emulates the proposed system (1) is described in this section to show its feasibility. Fig. 5 depicts the design of the circuit that emulates system (1). This circuit has three integrators (U1A, U2A, U3A), two inverting amplifiers (U4A, U5A) which are implemented with the operational amplifier TL082CD, as well as five signals multipliers (A1, A2, A3) by using the analog multiplier AD633.

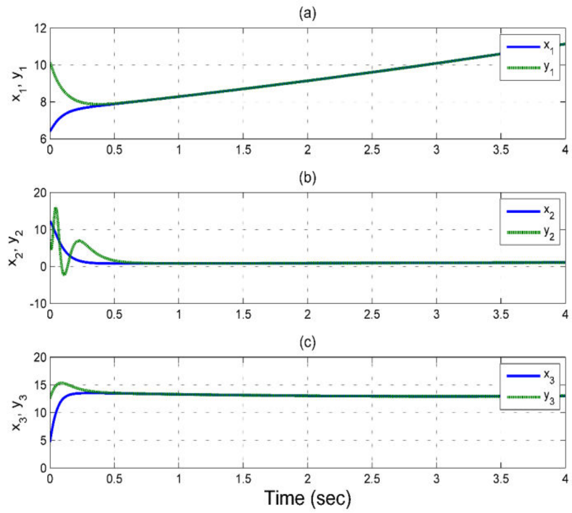


Fig. 3. Complete synchronization of the new chaotic systems

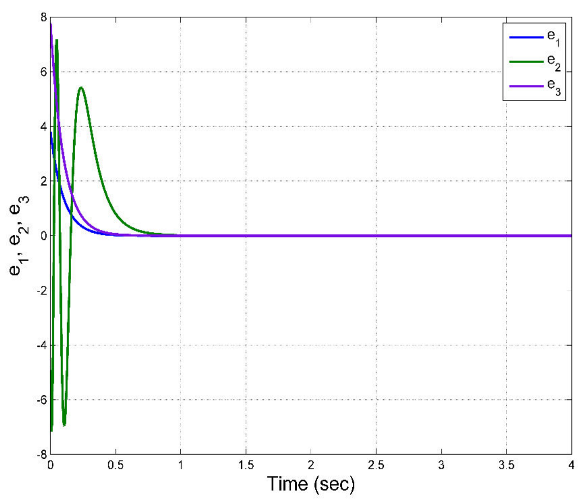


Fig. 4. Time-history of the synchronization errors for the new chaotic systems

The circuital equations of the designed circuit are given by

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1 R_1} x_2 \\ \dot{x}_2 = \dfrac{1}{10 C_2 R_2} x_1 x_3 - \dfrac{1}{C_2 R_3} x_2 - \dfrac{1}{10 C_2 R_4} x_2 x_3 \\ \dot{x}_3 = \dfrac{1}{10 C_3 R_5} x_2{}^2 - \dfrac{1}{C_3 R_6} V_1 \end{cases} \quad (26)$$

We choose the values of the circuital elements as

$$\begin{cases} R_2 = 200K\Omega, R_3 = 100k\Omega, \\ R_4 = R_5 = 20K\Omega, R_6 = 5k\Omega \\ R_1 = R_8 = R_9 = R_{10} = 10k\Omega \\ C_1 = C_2 = C_3 = 10nF \end{cases} \quad (27)$$

Where x_1, x_2, x_3 are corresponding the voltages at the capacitors (VC_1, VC_2, VC_3). As a result, it is easy to verify that the dimensionless system (26) corresponds to the introduced system with hidden attractor (1). The power supplies of all active devices are $\pm 15V_{DC}$. The proposed circuit is implemented by using the electronic simulation package MultiSIM. Figs 6-8 show the obtained phase portraits in (x_1, x_2) plane, (x_2, x_3) plane and (x_1, x_3) plane, respectively. There is a good agreement between these circuit simulation and numerical simulation (see Fig. 1).

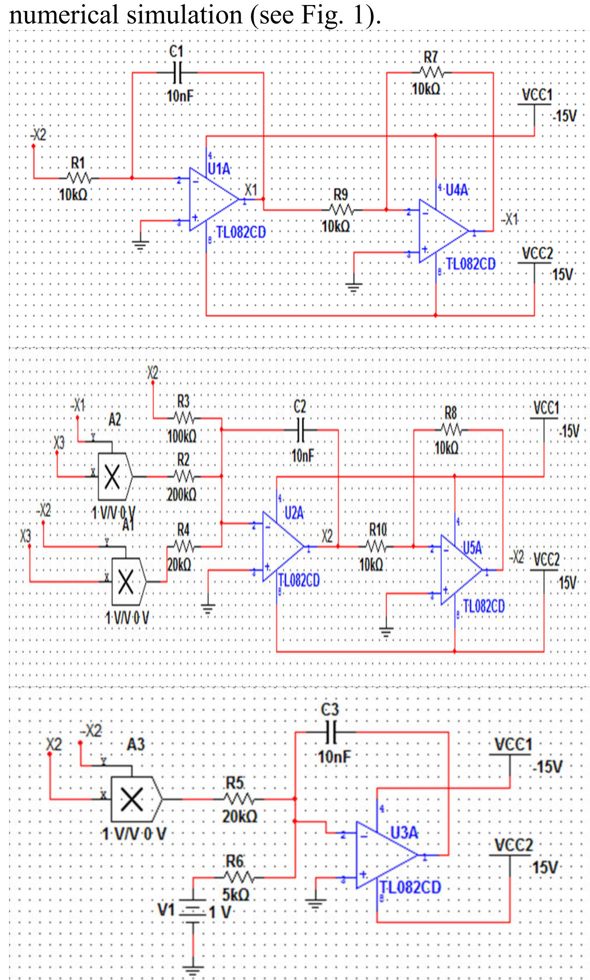


Fig. 5 Schematic of the proposed new chaotic system by using MultiSIM

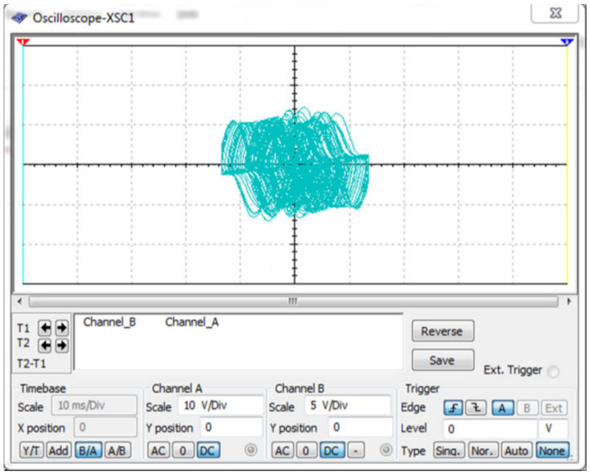


Fig. 6 2-D projection of the new chaotic system on the (x_1, x_2) plan

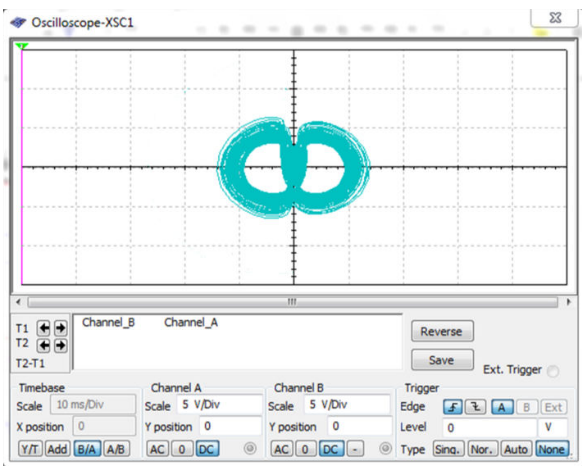


Fig. 7 2-D projection of the new chaotic system on the (x_2, x_3) plane

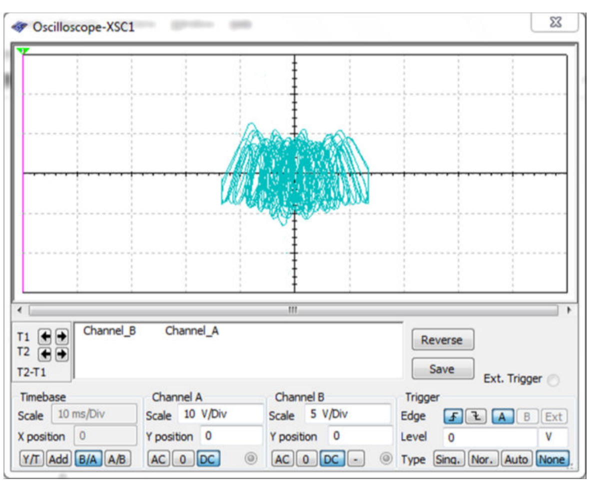


Fig. 8 2-D projection of the new chaotic system on the (x_1, x_3) plane

5 Conclusion

In this work, we presented a new chaotic system with hidden attractor. Excitingly, there is no equilibrium in this system and the system shows hidden chaotic attractors. Simulation results by using phase portraits, and Lyapunov exponents confirmed the new system's chaotic behavior. In addition, the possibility of synchronization of identical new chaotic systems with no equilibrium point has been analysed and confirmed. Finally, the feasibility of the theoretical model is also confirmed by an electronic circuitry.

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